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(54) **SYSTEM ESTIMATION METHOD, PROGRAM, RECORDING MEDIUM, SYSTEM ESTIMATION DEVICE**

SYSTEMSCHÄTZVERFAHREN, PROGRAMM, AUFZEICHNUNGSMEDIUM, SYSTEMSCHÄTZEINRICHTUNG

PROCEDE ET PROGRAMME D’EVALUATION DE SYSTEME, SUPPORT D’ENREGISTREMENT, DISPOSITIF D’EVALUATION DE SYSTEME

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- **NISHIYAMA K: "An H>tex<$_infty$>/ tex<Optimization and Its Fast Algorithm for Time-Variant System Identification", IEEE TRANSACTIONS ON SIGNAL PROCESSING, IEEE SERVICE CENTER, NEW YORK, NY, US, vol. 52, no. 5, 1 May 2004 (2004-05-01), pages 1335-1342, XP011110710, ISSN: 1053-587X, DOI: DOI:10.1109/TSP.2004.826156**
- **NISHIYAMA K: "FAST J-UNITARY ARRAY FORM OF THE HYPER H FILTER", IEICE TRANSACTIONS ON FUNDAMENTALS OF ELECTRONICS,COMMUNICATIONS AND COMPUTER SCIENCES, ENGINEERING SCIENCES SOCIETY, TOKYO, JP, vol. E88-A, no. 11, 1 November 2005 (2005-11-01), pages 3143-3150, XP001236658, ISSN: 0916-8508, DOI: 10.1093/IETFEC/E88-A.11.3143**
- **NISHIYAMA, K.: ’Robust Estimation of a Single Complex Sinusoid in White Noise-H Filtering Approach’ IEEE TRANSACTIONS ON SIGNAL PROCESSING, USA vol. 47, 1999, pages 2853 - 2856, XP002904274**

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- **NISHIYAMA, K, ET AL: 'H-learning of layered neural networks' IEEE TRANSACTIONS ON NEURAL NETWORKS, USA vol. 12, 06 November 2001, pages 1265 - 1277, XP002904275**

Description

Technical Field

[0001] The present invention relates to a system estimation method and program, a recording medium, and a system estimation device, and particularly to a system estimation method and program, a recording medium, and a system estimation device, in which the generation of robustness in state estimation and the optimization of a forgetting factor are simultaneously realized by using a fast H_∞ filtering algorithm of a hyper H_∞ filter developed on the basis of an H_∞ evaluation criterion.

Background Art

[0002] In general, system estimation means estimating a parameter of a mathematical model (transfer function, impulse response, etc.) of an input/output relation of a system based on input/output data. Typical application examples include an echo canceller in international communication, an automatic equalizer in data communication, an echo canceller and sound field reproduction in a sound system, active noise control in a vehicle etc. and the like. For more information, see non-patent document 1: "DIGITAL SIGNAL PROCESSING HANDBOOK" 1993, The Institute of Electronics, Information and Communication Engineers, and the like.

(Basic Principle)

[0003] Fig. 8 shows an example of a structural view for system estimation (unknown system may be expressed by an IIR (Infinite Impulse Response) filter).

[0004] This system includes an unknown system 1 and an adaptive filter 2. The adaptive filter 2 includes an FIR digital filter 3 and an adaptive algorithm 4.

[0005] Hereinafter, an example of an output error method to identify the unknown system 1 will be described. Here, u_k denotes an input of the unknown system 1, d_k denotes an output of the system, which is a desired signal, and $d\^{}_k$ denotes an output of the filter. (Incidentally, "^" means an estimated value and should be placed directly above a character, however, it is placed at the upper right of the character for input convenience. The same applies hereinafter.).

[0006] Since an impulse response is generally used as a parameter of an unknown system, the adaptive filter adjusts a coefficient of the FIR digital filter 3 by the adaptive algorithm so as to minimize an evaluation error $e_k = d_k - d\^{}_k$ of the figure.

[0007] Besides, conventionally, a Kalman filter based on an update expression (Riccati equation) of an error covariance matrix has been widely used for the estimation of a parameter (state) of a system. The details are disclosed in non-patent document 2: S. Haykin: Adaptive filter theory, Prentice-Hall (1996) and the like.

[0008] Hereinafter, the basic principle of the Kalman filter will be described.

[0009] A minimum variance estimate value $x\^{}_{k|k}$ of a state x_k of a linear system expressed in a state space model as indicated by the following expression:

$$\mathbf{x}_{k+1} = \rho^{-1/2}\mathbf{x}_k, \quad \mathbf{y}_k = \mathrm{H}_k\mathbf{x}_k + \mathbf{v}_k \qquad (1)$$

is obtained by using an error covariance matrix $\Sigma\^{}_{k|k-1}$ of the state as follows.

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k\hat{x}_{k|k-1})$$

$$\hat{x}_{k+1|k} = \rho^{-\frac{1}{2}}\hat{x}_{k|k} \qquad (2)$$

$$K_k = \hat{\Sigma}_{k|k-1}H_k^T(\rho + H_k\hat{\Sigma}_{k|k-1}H_k^T)^{-1} \qquad (3)$$

$$\hat{\Sigma}_{k|k} = \hat{\Sigma}_{k|k-1} - K_kH_k\hat{\Sigma}_{k|k-1}$$

$$\hat{\Sigma}_{k+1|k} = \hat{\Sigma}_{k|k}/\rho \quad (4)$$

where,

$$\hat{x}_{0|-1} = 0, \quad \hat{\Sigma}_{0|-1} = \varepsilon_0 I, \quad \varepsilon_0 > 0 \quad (5)$$

x_k: State vector or simply a state; unknown and this is an object of estimation.
y_k: Observation signal; input of a filter and known.
H_k: Observation matrix; known.
v_k: Observation noise; unknown.
p: Forgetting factor; generally determined by trial and error. K_k: Filter gain; obtained from matrix $\Sigma\hat{}_{k|k-1}$.
$\Sigma\hat{}_{k|k}$: Corresponds to the covariance matrix of an error of $x\hat{}_{k|k}$; obtained by a Riccati equation.
$\Sigma\hat{}_{k+1|k}$: Corresponds to the covariance matrix of an error of $x\hat{}_{k+1|k}$; obtained by the Riccati equation.
$\Sigma\hat{}_{1|0}$: Corresponds to the covariance matrix in an initial state; although originally unknown, $\varepsilon_0 I$ is used for convenience.

[0010] The present inventor has already proposed a system identification algorithm by a fast H_∞ filter (see patent document 1). This is such that an H_∞ evaluation criterion is newly determined for system identification, and a fast algorithm for the hyper H_∞ filter based thereon is developed, while a fast time-varying system identification method based on this fast H_∞ filtering algorithm is proposed. The fast H_∞ filtering algorithm can track a time-varying system which changes rapidly with a computational complexity of O (N) per unit-time step. It matches perfectly with a fast Kalman filtering algorithm at the limit of the upper limit value. By the system identification as stated above, it is possible to realize the fast real-time identification and estimation of the time-invariant and time-varying systems.
[0011] Incidentally, with respect to methods normally known in the field of the system estimation, see, for example, non-patent documents 2 and 3.

(Applied Example to Echo Canceller)

[0012] In a long distance telephone circuit such as an international telephone, a four-wire circuit is used from the reason of signal amplification and the like. On the other hand, since a subscriber's circuit has a relatively short distance, a two-wire circuit is used.
[0013] Fig. 9 is an explanatory view concerning a communication system and an echo. A hybrid transformer as shown in the figure is introduced at a connection part between the two-wire circuit and the four-wire circuit, and impedance matching is performed. When the impedance matching is complete, a signal (sound) from a speaker B reaches only a speaker A. However, in general, it is difficult to realize the complete matching, and there occurs a phenomenon in which part of the received signal leaks to the four-wire circuit, and returns to the receiver (speaker A) after being amplified. This is an echo (echo). As a transmission distance becomes long (as a delay time becomes long), the influence of the echo becomes large, and the quality of a telephone call is remarkably deteriorated (in the pulse transmission, even in the case of short distance, the echo has a large influence on the deterioration of a telephone call).
[0014] Fig. 10 is a principle view of an echo canceller.
[0015] Then, as shown in the figure, the echo canceller (echo canceller) is introduced, an impulse response of an echo path is successively estimated by using a received signal which can be directly observed and an echo, and a pseudo-echo obtained by using it is subtracted from the actual echo to cancel the echo and to remove it.
[0016] The estimation of the impulse response of the echo path is performed so that the mean square error of a residual echo e_k becomes minimum. At this time, elements to interfere with the estimation of the echo path are circuit noise and a signal (sound) from the speaker A. In general, when two speakers simultaneously start to speak (double talk), the estimation of the impulse response is suspended. Besides, since the impulse response length of the hybrid transformer is about 50 [ms], when the sampling period is made 125 [μs], the order of the impulse response of the echo path becomes actually about 400.

Non-patent document 1
"DIGITAL SIGNAL PROCESSING HANDBOOK" 1993 The Institute of Electronics, Information and Communication Engineers
Non-patent document 2
S. Haykin: Adaptive filter theory, Prentice-Hall (1996)

Non-patent document 3
B. Hassibi, A. H. Sayed, and T. Kailath: "Indefinite-Quadratic Estimation and Control", SIAM (1996)
Patent document 1
JP-A-2002-135171

[0017] With respect to the prior art attention is drawn to NISHIYAMA K: "Derivation of A Fast Algorithm of Modified H∞ Filters", INDUSTRIAL ELECTRONICS SOCIETY, 2000. IECON 2000. 26TH ANNUAL CONFERENCE OF THE IEEE 22-28 OCT. 2000, PISCATAWAY, NJ, USA, IEEE, vol. 1, 22 October 2000 (2000-10-22), pages 462-467, XP010569662, ISBN: 978-0-7803-6456-1. According to this document it is known that the fast Kalman filter provides very quick convergence at the computational complexity of the same order that the LMS algorithm requires. Nevertheless, its performance is still unsatisfactory in system identification because the conventional fast Kalman filter fails to track time-varying impulse responses of FIR systems. The failure of tracking is due to the absence of system noise in the statespace model to be used. However, according to the derivation of the fast Kalman filter, it is difficult to theoretically introduce the term of system noise into the algorithm. From this document, to overcome the difficulties, a new fast filtering algorithm, called a fast H_∞ filter, is disclosed based on the H_∞ theory.

[0018] US 2004/059551 A1 relates to quick real-time identification and estimation of a time-non-varying or time-varying system. A H_∞ evaluation criterion is determined, a fast algorithm for a modified H_∞ filter based on the criterion is developed, and a quick time-varying system identifying method according to the fast H_∞ filtering algorithm is provided. By the fast H_∞ filtering algorithm, a time-varying system sharply varying can be traced with an amount of calculation O(N) per unit time step. The algorithm completely agrees with a fast Kalman filtering algorithm at the extreme of the upper limit value. If the estimate of impulse response is determined, a pseudo-echo is sequentially determined from the estimate and subtracted from the real echo to cancel the echo. Thus, an echo canceller is realized.

[0019] US 5, 734, 715 A discloses to estimate the response of a system to an input signal, on the one hand the input signal and on the other hand an observation signal, a component of which is said response to the input signal, are received, an error signal is determined by subtracting from the observation signal the input signal filtered by an identification filter representative of the response of said system, and the coefficients of the identification filter are adapted by taking into account the input signal and the error signal. If the adaptation takes into account an adaption stepsize μ_t, the latter is varied according to: μ_t =a/(c+d. $P2_t/P1_t$), where a, c and d denote positive constants, $P1_t$ denotes an estimate of the power of the input signal and $P2_t$ denotes an estimate of the power of the observation signal or of a disturbance component of said observation signal. The variable μ_t can also be employed to adapt the gains of an echo canceller with adaptive gains.

[0020] Further attention is drawn to NISHIYAMA K: "An H∞ Optimization and Its Fast Algorithm for Time-Variant System Identification", IEEE TRANSACTIONS ON SIGNAL PROCESSING, IEEE SERVICE CENTER, NEW YORK, NY, US, vol. 52, no. 5, 1 May 2004 (2004-05-01), pages 1335-1342, XP011110710, ISSN: 1053-587X, DOI: DOI:10.1109/TSP.2004.826156. From this document it is known that in some estimation or identification techniques, a forgetting factor ρ has been used to improve the tracking performance for time-varying systems. However, the value of ρ has been typically determined empirically, without any evidence of optimality. This open problem was solved using the framework of H_∞ optimization. The resultant H_∞ filter enables the forgetting factor ρ to be optimized through a process noise that is determined by the filter Riccati equation. This document further explains the previously derived H_∞ filter, giving an H_∞ interpretation of its tracking capability. Additionally, a fast algorithm of the H_∞ filter, called the fast H_∞ filter, is presented when the observation matrix has a shifting property. Finally, the effectiveness of the derived fast algorithm is illustrated for time-variant system identification using several computer simulations. The fast H_∞ filter is shown to outperform the well-known least-mean-square algorithm and the fast Kalman filter in convergence rate.

[0021] NISHIYAMA K: "FAST J-UNITARY ARRAY FORM OF THE HYPER H∞ FILTER", IEICE TRANSACTIONS ON FUNDAMENTALS OF ELECTRONICS, COMMUNICATIONS AND COMPUTER SCIENCES, ENGINEERING SCIENCES SOCIETY, TOKYO, JP, vol. E88-A, no. 11, 1 November 2005 (2005-11-01), pages 3143-3150, XP001236658, ISSN: 0916-8508, DOI: 10.1093/IETFEC/E88-A.11.3143 starts from the hyper H_∞ filter for tracking of unknown time-varying systems. Additionally, a fast algorithm, called the fast H_∞ filter, of the hyper H_∞ filter is derived on condition that the observation matrix has a shifting property. This algorithm has a computational complexity of $\boldsymbol{O}(N)$ where N is the dimension of the state vector. However, there still remains a possibility of deriving alternative forms of the hyper H_∞ filter. In this document, a fast J-unitary form of the hyper H_∞ filter is derived, providing a new H_∞ fast algorithm, called the J-fast H_∞ filter. The J-fast H_∞ filter possesses a computational complexity of $\boldsymbol{O}(N)$, and the resulting algorithm is very amenable to parallel processing.

Disclosure of the Invention

[0022] However, in the conventional Kalman filter including the forgetting factor ρ as in the expressions (1) to (5), the value of the forgetting factor ρ must be determined by trial and error and a very long time has been required. Further, there has been no means for judging whether the determined value of the forgetting factor ρ is an optimal value.

[0023] Besides, with respect to the error covariance matrix used in the Kalman filter, it is known that a quadratic form to an arbitrary vector, which is originally not zero, is always positive (hereinafter referred to as "positive definite"), however, in the case where calculation is performed by a computer at single precision, the quadratic form becomes negative (hereinafter referred to as "negative definite"), and becomes numerically unstable. Besides, since the amount of calculation is $O(N^2)$ (or $O(N^3)$), in the case where the dimension N of the state vector x_k is large, the number of times of arithmetic operation per time step is rapidly increased, and it has not been suitable for a real-time processing.

[0024] In view of the above, the present invention has an object to establish an estimation method which can theoretically optimally determine a forgetting factor, and to develop an estimation algorithm and a fast algorithm which are numerically stable. Besides, the invention has an object to provide a system estimation method which can be applied to an echo canceller in a communication system or a sound system, sound field reproduction, noise control and the like.

[0025] In order to solve the problem, according to the invention, a newly devised H_∞ optimization method is used to derive a state estimation algorithm in which a forgetting factor can be optimally determined. Further, instead of an error covariance matrix which should always have the positive definite, its factor matrix is updated, so that an estimation algorithm and a fast algorithm, which are numerically stable, are developed.

[0026] The present invention relates to a system estimation method as defined in claim 1. Further, the present invention relates to a system estimation program as defined in claim 4 and to a computer readable recording medium as defined in claim 5. Finally, the present invention relates to a system estimation device as defined in claim 6.

[0027] Preferred embodiments of the invention are disclosed in the dependent claims.

[0028] According to the estimation method of the invention, the forgetting factor can be optimally determined, and the algorithm can stably operate even in the case of single precision, and accordingly, high performance can be realized at low cost. In general, in a normal civil communication equipment, calculation is often performed at single precision in view of cost and speed. Thus, as the practical state estimation algorithm, the invention would have effects in various industrial fields.

Brief Description of the Drawings

[0029]

Fig. 1 is a structural view of hardware of an embodiment.

Fig. 2 is a flowchart concerning the generation of robustness of an H_∞ filter and the optimization of a forgetting factor p.

Fig. 3 is a flowchart of an algorithm of the H_∞ filter (S105) in Fig. 2.

Fig. 4 is an explanatory view of a square root array algorithm of Theorem 2.

Fig. 5 is a flowchart of a fast algorithm of Theorem 3, which is numerically stable.

Fig. 6 is a view showing values of an impulse response $\{h_i\}_{i=0}^{23}$.

Fig. 7 shows an estimation result of the impulse response by the fast algorithm of Theorem 3, which is numerically stable.

Fig. 8 is a structural view for system estimation.

Fig. 9 is an explanatory view of a communication system and an echo.

Fig. 10 is a principle view of an echo canceller.

DESCRIPTION OF THE PREFERED EMBODIMENTS

[0030] Hereinafter, embodiments of the invention will be described.

1. Explanation of Symbols

[0031] First, main symbols used in the embodiments of the invention and whether they are known or unknown will be described.

x_k: State vector or simply a state; unknown and this is an object of the estimation.

x_o: Initial state; unknown.

w_k: System noise; unknown.

v_k: Observation noise; unknown.

y_k: Observation signal; input of a filter and known.

z_k: Output signal; unknown.

F_k: Dynamics of a system; known.

G_k: Drive matrix; known at the time of execution.

H_k: Observation matrix; known.
$x^\wedge_{k|k}$: Estimated value of a state x_k at a time k, using observation signals y_0 to y_k; given by a filter equation.
$x^\wedge_{k+1|k}$: Estimated value of a state x_{k+1} at a time k+1 using the observation symbols y_0 to y_k; given by the filter equation.
$x^\wedge_{0|0}$: Initial estimated value of a state; originally unknown, however, 0 is used for convenience.
$\Sigma^\wedge_{k|k}$: Corresponds to a covariance matrix of an error of $x^\wedge_{k|k}$; given by a Riccati equation.
$\Sigma^\wedge_{k+1|k}$: Corresponds to a covariance matrix of an error of $x^\wedge_{k+1|k}$; given by the Riccati equation.
$\Sigma^\wedge_{1|0}$: Corresponds to a covariance matrix in an initial state; originally unknown, however, $\varepsilon_0 I$ is used for convenience.
$K_{s,k}$: Filter gain; obtained from a matrix $\Sigma^\wedge_{k|k-1}$.
p: Forgetting factor; in the case of Theorems 1 to 3, when γ_f is determined, it is automatically determined by $\rho = 1 - \chi(\gamma_f)$.
$e_{f,i}$: Filter error
$R_{e,k}$: Auxiliary variable

[0032] Incidentally, "^" and "v" placed above the symbol mean estimated values. Besides, "~" , "-" , "U" and the like are symbols added for convenience. Although these symbols are placed at the upper right of characters for input convenience, as indicated in mathematical expressions, they are the same as those placed directly above the characters. Besides, x, w, H, G, K, R, Σ and the like are matrixes and should be expressed by thick letters as indicated in the mathematical expressions, however, they are expressed in normal letters for input convenience.

2. Hardware and Program for System Estimation

[0033] The system estimation or the system estimation device and system can be provided by a system estimation program for causing a computer to execute respective procedures, a computer readable recording medium recording the system estimation program, a program product including the system estimation program and capable of being loaded into an internal memory of a computer, a computer, such as a server, including the program, and the like.
[0034] Fig. 1 is a structural view of hardware of this embodiment.
[0035] This hardware includes a processing section 101 which is a central processing unit (CPU), an input section 102, an output section 103, a display section 104 and a storage section 105. Besides, the processing section 101, the input section 102, the output section 103, the display section 104 and the storage section 105 are connected by suitable connection means such as a star or a bus. Known data indicated in "1. Explanation of Symbols" and subjected to the system estimation are stored in the storage section 105 as the need arises. Besides, unknown and known data, calculated data relating to the hyper H_∞ filter, and other data are written and/or read by the processing section 101 as the need arises.

3. Hyper H_∞ Filter by Which Forgetting Factor Can Be Optimally Determined

(Theorem 1)

[0036] Consideration is given to a state space model as indicated by following expressions.

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_k\boldsymbol{x}_k + \boldsymbol{G}_k\boldsymbol{w}_k, \qquad \boldsymbol{w}_k, \boldsymbol{x}_k \in \mathcal{R}^N \tag{6}$$

$$y_k = \boldsymbol{H}_k\boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{7}$$

$$z_k = \boldsymbol{H}_k\boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N},\ k = 0, 1, \ldots, L \tag{8}$$

[0037] An H_∞ evaluation criterion as indicated by the following expression is proposed for the state space model as described above.

$$\sup_{x_0,\{w_i\},\{v_i\}} \frac{\sum_{i=0}^{k} \|e_{f,i}\|^2/\rho}{\|\boldsymbol{x}_0 - \check{\boldsymbol{x}}_{0|-1}\|^2_{\boldsymbol{\Sigma}_0^{-1}} + \sum_{i=0}^{k}\|\boldsymbol{w}_i\|^2 + \sum_{i=0}^{k}\|v_i\|^2/\rho} < \gamma_f^2 \tag{9}$$

[0038] A state estimated value $\hat{x}_{k|k}$ (or an output estimated value $\check{z}_{k|k}$) to satisfy this H_∞ evaluation criterion is given by a hyper H_∞ filter of level γ_f:

$$\check{z}_{k|k} = H_k \hat{x}_{k|k} \tag{10}$$

$$\hat{x}_{k|k} = F_{k-1}\hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k F_{k-1}\hat{x}_{k-1|k-1}) \tag{11}$$

$$K_{s,k} = \hat{\Sigma}_{k|k-1} H_k^T (H_k \hat{\Sigma}_{k|k-1} H_k^T + \rho)^{-1} \tag{12}$$

$$\left.\begin{aligned} \hat{\Sigma}_{k|k} &= \hat{\Sigma}_{k|k-1} - \hat{\Sigma}_{k|k-1} C_k^T R_{e,k}^{-1} C_k \hat{\Sigma}_{k|k-1} \\ \hat{\Sigma}_{k+1|k} &= \left(F_k \hat{\Sigma}_{k|k} F_k^T\right)/\rho \end{aligned}\right\} \tag{13}$$

where,

$$e_{f,i} = \check{z}_{i|i} - H_i x_i, \quad \hat{x}_{0|0} = \check{x}_0, \quad \hat{\Sigma}_{1|0} = \Sigma_0$$

$$R_{e,k} = R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad R_k = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix} \tag{14}$$

$$0 < \rho = 1 - \chi(\gamma_f) \leq 1, \quad \gamma_f > 1 \tag{15}$$

Incidentally, expression (11) denotes a filter equation, expression (12) denotes a filter gain, and expression (13) denotes a Riccati equation.

[0039] Besides, a drive matrix G_k is generated as follows.

$$G_k G_k^T = \frac{\chi(\gamma_f)}{\rho} F_k \hat{\Sigma}_{k|k} F_k^T \tag{16}$$

[0040] Besides, in order to improve the tracking capacity of the foregoing H_∞ filter, the upper limit value γ_f is set to be as small as possible so as to satisfy the following existence condition.

$$\hat{\Sigma}_{i|i}^{-1} = \hat{\Sigma}_{i|i-1}^{-1} + \frac{1-\gamma_f^{-2}}{\rho} H_i^T H_i > 0, \quad i = 0, \ldots, k \tag{17}$$

Where, $\chi(\gamma_f)$ is a monotonically damping function of γ_f, which satisfies $\chi(1) = 1$ and $\chi(\infty) = 0$.

[0041] The feature of Theorem 1 is that the generation of robustness in the state estimation and the optimization of the forgetting factor ρ are simultaneously performed.

[0042] Fig. 2 shows a flowchart concerning the generation of robustness of the H_∞ filter and the optimization of the forgetting factor p. Here,
block "EXC > 0": an existence condition of the H_∞ filter, and $\Delta\gamma$: a positive real number.

[0043] First, the processing section 101 reads out or inputs the upper limit value γ_f from the storage section 105 or the input section 102 (S101). In this example, $\gamma_f >> 1$ is given. The processing section 101 determines the forgetting factor ρ by expression (15) (S103). Thereafter, the processing section 101 executes the hyper H_∞ filter of expression (10) to expression (13) based on the forgetting factor ρ (S105). The processing section 101 calculates expression (17) (or the right side (this is made EXC) of after-mentioned expression (18)) (S107), and when the existence condition is satisfied at all times (S109), γ_f is decreased by $\Delta\gamma$, and the same processing is repeated (S111). On the other hand, when the existence condition is not satisfied at a certain γ_f (S109), what is obtained by adding $\Delta\gamma$ to the γ_f is made the optimal value γ_f^{op} of γ_f, and is outputted to the output section 103 and/or stored into the storage section 105 (S113). Incidentally, in this example, although $\Delta\gamma$ is added, a previously set value other than that may be added. This optimization process is called a y-iteration. Incidentally, the processing section 101 may store a suitable intermediate value and a final value obtained at respective steps, such as the H_∞ filter calculation step S105 and the existence condition calculation step S107, into the storage section 105 as the need arises, and may read them from the storage section 105.

[0044] When the hyper H_∞ filter satisfies the existence condition, the inequality of expression (9) is always satisfied. Thus, in the case where the disturbance energy of the denominator of expression (9) is limited, the total sum of the square estimated error of the numerator of expression (9) becomes bounded, and the estimated error after a certain time becomes 0. This means that when γ_f can be made smaller, the estimated value $x\hat{}_{k|k}$ can quickly follow the change of the state x_k.

[0045] Here, attention should be given to the fact that the algorithm of the hyper H_∞ filter of Theorem 1 is different from that of the normal H_∞ filter. Besides, when $\gamma_f \to \infty$, then $\rho = 1$ and $G_k = 0$, and the algorithm of the H_∞ filter of Theorem 1 coincides with the algorithm of the Kalman filter.

[0046] Fig. 3 is a flowchart of the algorithm of the (hyper) H_∞ filter (S105) in Fig. 2.

[0047] The hyper H_∞ filtering algorithm can be summarized as follows.

[0048] [Step S201] The processing section 101 reads out the initial condition of a recursive expression from the storage section 105 or inputs the initial condition from the input section 102, and determines it as indicated in the figure. Incidentally, L denotes a previously fixed maximum data number.

[0049] [Step S203] The processing section 101 compares the time k with the maximum data number L. When the time k is larger than the maximum data number, the processing section 101 ends the processing, and when not larger, advance is made to a next step. (If unnecessary, the conditional sentence can be removed. Alternatively, restart may be made as the need arises.)

[0050] [Step S205] The processing section 101 calculates a filter gain $K_{s,k}$ by using expression (12).

[0051] [Step S207] The processing section 101 updates the filter equation of the hyper H_∞ filter of expression (11).

[0052] [Step S209] The processing section 101 calculates terms $\Sigma\hat{}_{k|k}$, $\Sigma\hat{}_{k+1|k}$ corresponding to the covariance matrix of an error by using the Riccati equation of expression (13).

[0053] [Step S211] The time k is made to advance (k = k + 1), return is made to step S203, and continuation is made as long as data exists.

[0054] Incidentally, the processing section 101 may store a suitable intermediate value, a final value, a value of the existence condition and the like obtained at the respective steps, such as the H_∞ filter calculation steps S205 to S209, into the storage section 105 as the need arises, or may read them from the storage section 105.

(Scalar Existence Condition)

[0055] The amount of calculation $O(N^2)$ was necessary for the judgment of the existence condition of expression (17). However, when the following condition is used, the existence of the H_∞ filter of Theorem 1, that is, expression (9) can be verified by the amount of calculation O(N).

Corollary 1: Scalar existence condition

[0056] When the following existence condition is used, the existence of the hyper H_∞ filter can be judged by the amount of calculation O(N).

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \tag{18}$$

Here;

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1 - H_i K_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \tag{19}$$

Where, $K_{s,i}$ denotes the filter gain obtained in expression (12).

(Proof)

[0057] Hereinafter, the proof of the system 1 will be described.

[0058] When a characteristic equation

$$|\lambda I - R_{e,k}| = \begin{vmatrix} \lambda - (\rho + H_k\hat{\Sigma}_{k|k-1}H_k^T) & -H_k\hat{\Sigma}_{k|k-1}H_k^T \\ -H_k\hat{\Sigma}_{k|k-1}H_k^T & \lambda - (-\rho\gamma_f^2 + H_k\hat{\Sigma}_{k|k-1}H_k^T) \end{vmatrix}$$

$$= \lambda^2 - (2H_k\hat{\Sigma}_{k|k-1}H_k^T + \rho\varrho)\lambda - \rho^2\gamma_f^2 + \rho\varrho H_k\hat{\Sigma}_{k|k-1}H_k^T = 0$$

of a 2 × 2 matrix $R_{e,k}$ is solved, an eigenvalue λ_i of $R_{e,k}$ is obtained as follows.

$$\lambda_i = \frac{\Phi \pm \sqrt{\Phi^2 - 4\rho\varrho H_k\hat{\Sigma}_{k|k-1}H_k^T + 4\rho^2\gamma_f^2}}{2}$$

[0059] Where,

$$\Phi = 2H_k\hat{\Sigma}_{k|k-1}H_k^T + \rho\varrho,\ \varrho = 1 - \gamma_f^2$$

$$-4\rho\varrho H_k\hat{\Sigma}_{k|k-1}H_k^T + 4\rho^2\gamma_f^2 > 0$$

one of two eigenvalues of the matrix $R_{e,k}$ becomes positive, the other becomes negative, and the matrixes R_k and $R_{e,k}$ have the same inertia. By this, when

$$H_k\hat{\Sigma}_{k|k-1}H_k^T = \frac{H_k\tilde{K}_k}{1 - \frac{1-\gamma_f^{-2}}{\rho}H_k\tilde{K}_k},\quad H_k\tilde{K}_k = \frac{\rho H_k K_{s,k}}{1-\gamma_f^{-2}H_k K_{s,k}}$$

is used, the existence condition of expression (18) is obtained. Here, the amount of calculation of $H_kK_{s,k}$ is O(N).

4. State Estimation Algorithm Which Is Numerically Stable

[0060] Since the foregoing hyper H_∞ filter updates $\Sigma\hat{}_{k|k-1} \in R^{n\times n}$, the amount of calculation per unit time step becomes $O(N^2)$, that is, an arithmetic operation proportional to N^2 becomes necessary. Here, N denotes the dimension of the state vector x_k. Thus, as the dimension of x_k is increased, the calculation time required for execution of this filter is rapidly increased. Besides, although the error covariance matrix $\Sigma\hat{}_{k|k-1}$ must always have the positive definite from its property, there is a case where it has numerically the negative definite. Especially, in the case where calculation is made at single precision, this tendency becomes remarkable. At this time, it is known that the filter becomes unstable. Thus, in order to put the algorithm to practical use and to reduce the cost, the development of the state estimation algorithm which can be operated even at single precision (example: 32 bit) is desired.

[0061] Then, next, attention is paid to

$$R_k = R^{1/2}{}_k J_1 R^{1/2}{}_k,$$

$$R_{e,k} = R^{1/2}{}_{e,k} J_1 R^{1/2}{}_{e,k},$$

and

$$\Sigma\hat{}_{k|k-1} = \Sigma\hat{}^{1/2}_{k|k-1}\Sigma\hat{}^{1/2}_{k|k-1},$$

and an H_∞ filter (square root array algorithm) of Theorem 1, which is numerically stabilized, is indicated in Theorem 2. Here, although it is assumed that $F_k = I$ is established for simplification, it can be obtained in the same way also in the case of $F_k \neq I$. Hereinafter, the hyper H_∞ filter to realize the state estimation algorithm which is numerically stable will be indicted.

(Theorem 2)

[0062]

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k\hat{x}_{k-1|k-1}) \tag{20}$$

$$K_{s,k} = K_k(:,1)/R_{e,k}(1,1)\ ,\ K_k = \rho^{\frac{1}{2}}(\rho^{-\frac{1}{2}}K_kR_{e,k}^{-\frac{1}{2}}J_1^{-1})J_1R_{e,k}^{\frac{1}{2}} \tag{21}$$

$$\left[\begin{array}{c|c} R_k^{\frac{1}{2}} & C_k\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ \hline 0 & \rho^{-\frac{1}{2}}\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{array}\right]\Theta(k) = \left[\begin{array}{c|c} R_{e,k}^{\frac{1}{2}} & 0 \\ \hline \rho^{-\frac{1}{2}}K_kR_{e,k}^{-\frac{1}{2}}J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{array}\right] \tag{22}$$

Where,

$$R_k = R_k^{\frac{1}{2}}J_1R_k^{\frac{1}{2}},\ R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}}\gamma_f \end{bmatrix},\ J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}}\hat{\Sigma}_{k|k-1}^{\frac{1}{2}}$$

$$R_{e,k} = R_k + C_k\hat{\Sigma}_{k|k-1}C_k^T,\ C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix},\ R_{e,k} = R_{e,k}^{\frac{1}{2}}J_1R_{e,k}^{\frac{1}{2}},\ \hat{x}_{0|0} = \check{x}_0 \tag{23}$$

$\Theta(k)$ denotes a J-unitary matrix, that is, $\Theta(k)J\Theta(k)^T = J$ is satisfied, $J = (J_1 \oplus I)$, and I is a unit matrix. Besides, $K_k(:,1)$ indicates the column vector of the first column of the matrix K_k.

[0063] Incidentally, in expressions (21) and (22), J_1^{-1} and J_1 can be deleted.

[0064] Fig. 4 is an explanatory view of the square root array algorithm of Theorem 2. This calculation algorithm can be used in the calculation (S105) of the H_∞ filter in the flowchart of Theorem 1 shown in Fig. 2.

[0065] In this estimation algorithm, instead of obtaining $\Sigma\hat{}_{k|k-1}$ by a Riccati type update expression, its factor matrix $\Sigma\hat{}^{1/2}_{k|k-1} \in R^{N\times N}$ (square root matrix of $\Sigma\hat{}_{k|k-1}$) is obtained by the update expression based on the J-unitary transformation. From a 1-1 block matrix and a 2-1 block matrix generated at this time, the filter gain $K_{s,k}$ is obtained as shown in the figure. Thus, $\Sigma\hat{}_{k|k-1} = \Sigma\hat{}^{1/2}_{k|k-1}\ \Sigma\hat{}^{1/2}_{k|k-1} > 0$ is established, the positive definite property of $\Sigma\hat{}_{k|k-1}$ is ensured, and it can be numerically stabilized. Incidentally, a computational complexity of the H_∞ filter of Theorem 2 per unit step remains $O(N^2)$.

[0066] Incidentally, in Fig. 4, J_1^{-1} can be deleted.

[0067] First, the processing section 101 reads out terms contained in the respective elements of the left-side equations of expression (22) from the storage section 105 or obtains them from the internal memory or the like, and executes the J-unitary transformation (S301). The processing section 101 calculates system gains K_k and $K_{s,k}$ from the elements of the right-side equations of the obtained expression (22) based on expression (21) (S303, S305). The processing section 101 calculates the state estimated value $x\hat{}_{k|k}$ based on expression (20) (S307).

5. Numerically Stable Fast Algorithm for State Estimation

[0068] As described above, a computational complexity of the H_∞ filter of Theorem 2 per unit step remains $O(N^2)$. Then, as a countermeasure for the complexity, by using that when $\underline{H}_k = \underline{H}_{k+1}\Psi$, $\underline{H}_k = [u(k), \cdots, u(0), 0, \cdots, 0]$, a covariance matrix $\Sigma_{k+1k|k}$ of one step prediction error of $\underline{x}_k = [x^T_k, 0^T]^T$ satisfies

$$\underline{\Sigma}_{k+1|k} - \Psi \underline{\Sigma}_{k|k-1} \Psi^T = -\underline{L}_k R_{r,k}^{-1} \underline{L}_k^T, \quad \underline{L}_k = \begin{bmatrix} \tilde{L}_k \\ 0 \end{bmatrix} \tag{24}$$

consideration is given to updating $\underline{L}_k$ (that is, $L^{\sim}_k$) with a low dimension instead of $\underline{\Sigma}_{k+1|k}$. Here, when attention is paid to $R_{r,k} = R^{1/2}_{r,k} SR^{1/2}_{r,k}$, next Theorem 3 can be obtained.

(Theorem 3)

[0069]

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + \overline{K}_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \tag{61}$$

$$\overline{K}_{s,k} = \overline{K}_k(:,1)/R_{e,k}(1,1) \ , \ \overline{K}_k = \rho^{\frac{1}{2}}(\overline{K}_k R_{e,k}^{-\frac{1}{2}}) R_{e,k}^{\frac{1}{2}} \tag{62}$$

$$\begin{bmatrix} R_{e,k+1}^{\frac{1}{2}} & 0 \\ \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} R_{e,k+1}^{-\frac{1}{2}} J_1 & \tilde{L}_{k+1} R_{r,k+1}^{-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-\frac{1}{2}} \\ \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-\frac{1}{2}} J_1 & \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-\frac{1}{2}} \end{bmatrix} \Theta(k) \tag{63}$$

Here, $\Theta(k)$ denotes an arbitrary J-unitary matrix, and $C_k = C_{k+1}\Psi$ is established.
Where,

$$R_k = R_k^{\frac{1}{2}} J_1 R_k^{\frac{1}{2}}, \quad R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}}\gamma_f \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}}$$

$$R_{e,k} = R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_1 R_{e,k}^{\frac{1}{2}}, \quad \hat{x}_{0|0} = \check{x}_0 \tag{23}$$

[0070] Incidentally, the proof of Theorem 3 will be described later.

[0071] The above expression can be arranged with respect to K_k instead of $K^{\sim}_k (= P^{-1/2} K_k)$.

[0072] Further, when the following J-unitary matrix

$$\Theta(k) = (J_1 R_{e,k}^{\frac{1}{2}} \oplus -R_{r,k}^{\frac{1}{2}}) \Sigma(k) (R_{e,k+1}^{-\frac{1}{2}} J_1^{-1} \oplus -R_{r,k+1}^{-\frac{1}{2}})$$

is used, a fast state estimation algorithm of Theorem 4 can be obtained. Where, Ψ denotes a shift matrix.

(Theorem 4)

[0073]

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \tag{25}$$

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1)/R_{e,k}(1,1) \tag{26}$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (30)$$

Where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}} K_k \quad (31)$$

dig[·] denotes a diagonal matrix, and $R_{e,k+1}$ (1,1) denotes a 1-1 element of the matrix $R_{e,k+1}$. Besides, the above expression can be arranged with respect to K_k instead of $K^{\sim}_k$.

[0074] In the fast algorithm, since the filter gain $K_{s,k}$ is obtained by the update of $L^{\sim}_k \in R^{(N+1)\times 2}$ in the following factoring

$$\underline{\Sigma}_{k+1|k} - \Psi \underline{\Sigma}_{k|k-1} \Psi^T = -\underline{L}_k R_{r,k}^{-1} \underline{L}_k^T \quad (32)$$

O(N+1) is sufficient for the amount of calculation per unit step.
Here, attention should be paid to the following expression.

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} = \rho^{-\frac{1}{2}} \left(\underline{\Sigma}_{k+1|k} \check{C}_{k+1}^T - \Psi \underline{\Sigma}_{k|k-1} \check{C}_k^T \right)$$

[0075] Fig. 5 is a flowchart of a numerically stable fast algorithm of Theorem 4. The fast algorithm is incorporated in the calculation step (S105) of the H_∞ filter of Fig. 2, and is optimized by the γ-iteration. Thus, during a period in which the existence condition is satisfied, γ_f is gradually decreased, however, at the time point when it comes to be unsatisfied, γ_i is increased as indicated in the figure.

[0076] The H_∞ filtering algorithm can be summarized as follows.

[0077] [Step S401] The processing section 101 determines an initial condition of the recursive expression as indicated in the figure. Incidentally, L denotes a maximum data number.

[0078] [Step S403] The processing section 101 compares the time k with the maximum data number L. When the time k is larger than the maximum data number, the processing section 101 ends the processing, and when not larger, advance is made to a next step. (When unnecessary, the conditional sentence can be removed. Alternatively, restart is made.)

[0079] [Step S405] The processing section 101 recursively calculates a term K_{k+1} corresponding to a filter gain by using expressions (27) and (31).

[0080] [Step S406] The processing section 101 recursively calculates $R_{e,k+1}$ by using expression (29).

[0081] [Step S407] The processing section 101 further calculates $K_{s,k}$ by using expressions (26) and (31).

[0082] [Step S409] The processing section 101 judges the existence condition EXC > 0 here, and when the existence condition is satisfied, advance is made to step S411.

[0083] [Step S413] On the other hand, when the existence condition is not satisfied at step S409, the processing section 101 increases γ_f, and return is made to step S401.

[0084] [Step S411] The processing section 101 updates the filter equation of the H_∞ filter of expression (25).

[0085] [Step S415] The processing section 101 recursively calculates $R_{r,k+1}$ by using expression (30). Besides, the processing section 101 recursively calculates $L^{\sim}_{k+1}$ by using expressions (28) and (31).

[0086] [Step S419] The processing section 101 advances the time k (k = k+1), returns to step S403, and continues as long as data exists.

[0087] Incidentally, the processing section 101 may store a suitable intermediate value and a final value obtained at the respective steps, such as the H_∞ filter calculation steps S405 to S415 and the calculation step S409 of the existence condition, into the storage section 105 as the need arises, and may read them from the storage section 105.

6. Echo Canceller

[0088] Next, a mathematical model of an echo canceling problem is generated.

[0089] First, when consideration is given to the fact that a received signal $\{u_k\}$ becomes an input signal to an echo path, by a (time-varying) impulse response $\{h_i[k]\}$ of the echo path, an observed value $\{y_k\}$ of an echo $\{d_k\}$ is expressed by the following expression.

$$y_k = d_k + v_k = \sum_{i=0}^{N-1} h_i[k]u_{k-i} + v_k, \quad k = 0, 1, 2, \cdots \tag{33}$$

Here, u_k and y_k respectively denote the received signal and the echo at a time t_k (= kT; T is a sampling period), v_k denotes circuit noise having a mean value of 0 at the time t_k, $h_i[k]$, i = 0, $\cdots$, N-1 denotes a time-varying impulse response, and the tap number N thereof is known. At this time, when estimated values $\{\hat{h}_i[k]\}$ of the impulse response are obtained every moment, a pseudo-echo as indicated below can be generated by using that.

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k]u_{k-i}, \quad k = 0, 1, 2, \cdots \tag{34}$$

When this is subtracted from the echo ($y_k - \hat{d}_k \approx 0$), the echo can be cancelled. Where, it is assumed that if k-i < 0, then $u_{k-1} = 0$.

[0090] From the above, the problem can be reduced to the problem of successively estimating the impulse response $\{h_i[k]\}$ of the echo path from the received signal $\{u_k\}$ and the echo $\{y_k\}$ which can be directly observed.

[0091] In general, in order to apply the H_∞ filter to the echo canceller, first, expression (32) must be expressed by a state space model including a state equation and an observation equation. Then, since the problem is to estimate the impulse response $\{h_i[k]\}$, when $\{h_i[k]\}$ is made a state variable x_k, and a variance of about w_k is allowed, the following state space model can be established for the echo path.

$$x_{k+1} = x_k + G_k w_k, \qquad x_k, w_k \in \mathcal{R}^N \tag{35}$$

$$y_k = H_k x_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{36}$$

$$z_k = H_k x_k, \qquad z_k \in \mathcal{R},\ H_k \in \mathcal{R}^{1\times N} \tag{37}$$

Where,

$$x_k = [h_0[k], \cdots, h_{N-1}[k]]^T, \quad w_k = [w_k(1), \cdots, w_k(N)]^T$$

$$H_k = [u_k, \cdots, u_{k-N+1}]$$

[0092] The hyper and fast H_∞ filtering algorithms to the state space model as stated above is as described before. Besides, at the estimation of the impulse response, when the generation of a transmission signal is detected, the estimation is generally suspended during that.

7. Evaluation to Impulse Response

(Confirmation of Operation)

[0093] With respect to the case where the impulse response of the echo pulse is temporally invariable ($h_i[k] = h_i$), and the tap number N is 48, the operation of the fast algorithm is confirmed by using a simulation.

$$y_k = \sum_{i=0}^{47} h_i u_{k-i} + v_k \qquad (38)$$

Incidentally, Fig. 6 is a view showing values of the impulse response $\{h_i\}$ here.

[0094] Here, the value shown in the figure are used for the impulse response $\{h_i\}_{i=0}^{23}$, and the other $\{h_i\}_{i=24}^{47}$ is made 0. Besides, it is assumed that v_k is stationary Gaussian white noise having a mean value of 0 and variance $\sigma_v^2 = 1.0 \times 10^{-6}$, and the sampling period T is made 1.0 for convenience.

[0095] Besides, the received signal $\{u_k\}$ is approximated by a secondary AR model as follows.

$$u_k = \alpha_1 u_{k-1} + \alpha_2 u_{k-2} + w_k' \qquad (39)$$

Where, $\alpha_1 = 0.7$ and $\alpha_2 = 0.1$ are assumed, and w_k' denotes stationary Gaussian white noise having a means value of 0 and variance $\sigma_{w'}^2 = 0.04$.

(Estimation Result of Impulse Response)

[0096] Fig. 7 shows an estimation result of the impulse response by the numerically stable fast algorithm of Theorem 3. Here, the vertical axis of Fig. 7(b) indicates

$$\sqrt{\{\sum_{i=0}^{47}(h_i - \hat{x}_k(i+1))^2\}}.$$

[0097] By this, it is understood that the estimation can be excellently performed by the fast algorithm. Where, $\rho = 1-\chi(\gamma_f)$, $\chi(\gamma_f) = \gamma_f^{-2}$, $\hat{x}_{0|0} = 0$ and $\hat{\Sigma}_{0|0} = 20I$ were assumed, and the calculation was performed at double precision. Besides, while the existence condition is confirmed, $\gamma_f = 5.5$ was set.

8. Proof of Theorem

8-1. Proof of Theorem 2

[0098] When the following expression:

$$\begin{bmatrix} R_k^{\frac{1}{2}} & C_k\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ 0 & \rho^{-\frac{1}{2}}\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{bmatrix} J \begin{bmatrix} R_k^{\frac{1}{2}} & 0 \\ \hat{\Sigma}_{k|k-1}^{\frac{1}{2}}C_k^T & \rho^{-\frac{1}{2}}\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & 0 \\ \rho^{-\frac{1}{2}}K_kR_{e,k}^{-\frac{1}{2}}J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{bmatrix} J \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & \rho^{-\frac{1}{2}}J_1^{-1}R_{e,k}^{-\frac{1}{2}}K_k^T \\ 0 & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{bmatrix} \quad (40)$$

is established, following expressions are obtained by comparing the respective terms of 2 × 2 block matrixes of both sides.

$$R_{e,k} = R_k + C_k\hat{\Sigma}_{k|k-1}C_k^T \quad (41)$$

$$K_k = \hat{\Sigma}_{k|k-1}C_k^T \quad (42)$$

$$\hat{\Sigma}_{k+1|k} + \rho^{-1}K_kR_{e,k}^{-1}K_k^T = \rho^{-1}\hat{\Sigma}_{k|k-1} \quad (43)$$

[0099] This is coincident with the Riccati equation of expression (13) at $F_k = I$ of Theorem 1. Where,

$$J = (J_1 \oplus I),\quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix} \quad (44)$$

[0100] On the other hand, when $AJA^T = BJB^T$ is established, B can be expressed as $B = A\Theta(k)$ by using the J-unitary matrix $\Theta(K)$. Thus, from expression (40), the Riccati equation of Theorem 1 is equivalent to the following expression.

$$\begin{bmatrix} R_{e,k}^{\frac{1}{2}} & 0 \\ \rho^{-\frac{1}{2}}K_kR_{e,k}^{-\frac{1}{2}}J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} R_k^{\frac{1}{2}} & C_k\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ 0 & \rho^{-\frac{1}{2}}\hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{bmatrix}\Theta(k) \quad (45)$$

[0101] Incidentally, in expressions (40) and (45), J_1^{-1} can be deleted.

8-2. Proof of Theorem 3

[0102] It is assumed that there is a J-unitary matrix $\Theta(k)$ which performs block triangulation as follows.

$$\begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix} = \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & \check{C}_{k+1}\tilde{L}_kR_{r,k}^{-\frac{1}{2}} \\ \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix}R_{e,k}^{-\frac{1}{2}}J_1 & \rho^{-\frac{1}{2}}\tilde{L}_kR_{r,k}^{-\frac{1}{2}} \end{bmatrix}\Theta(k).$$

At this time, when both sides $J = (J_1 \oplus -S)$ - norm of the above expression are compared, X, Y and Z of the left side can be determined as follows. Where, S denote a diagonal matrix in which diagonal elements take 1 or -1.

(1,1)-Block matrix

[0103]

$$
\begin{aligned}
& XJ_1X^T \\
&\quad = R_{e,k} - \breve{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T \\
&\quad = R_{e,k} + \breve{C}_{k+1}\left(\breve{\Sigma}_{k+1|k} - \Psi\breve{\Sigma}_{k|k-1}\Psi^T\right)\breve{C}_{k+1}^T \\
&\quad = R_{e,k} + \breve{C}_{k+1}\breve{\Sigma}_{k+1|k}\breve{C}_{k+1}^T - \breve{C}_k\breve{\Sigma}_{k|k-1}\breve{C}_k^T \\
&\quad = R_{e,k} + (R_{e,k+1} - R_{k+1}) - (R_{e,k} - R_k) = R_{e,k+1}
\end{aligned}
$$

Thus, $X = R_{e,k+1}^{\frac{1}{2}}$ is obtained from $R_{e,k+1} = R_{e,k+1}^{\frac{1}{2}} J_1 R_{e,k+1}^{\frac{1}{2}},\ R_{k+1} = R_k$. Here , attention should be paid to the fact that

$$
J_1^{-1} = J_1\ (J_1^2 = I),\ S^{-1} = S,\ R_{e,k+1}^T = R_{e,k+1},\ R_{r,k}^T = R_{r,k},\ R_{r,k}^{-1} = R_{r,k}^{-\frac{1}{2}} S R_{r,k}^{-\frac{1}{2}},
$$
$$
\breve{C}_k = \breve{C}_{k+1}\Psi\ (\breve{C}_k^T = \Psi^T\breve{C}_{k+1}^T)
$$

is established.

(2,1)-Block matrix

[0104]

$$
\begin{aligned}
& YJ_1X^T \\
&\quad = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T \\
&\quad = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} + \rho^{-\frac{1}{2}}\left(\breve{\Sigma}_{k+1|k} - \Psi\breve{\Sigma}_{k|k-1}\Psi^T\right)\breve{C}_{k+1}^T \\
&\quad = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} + \rho^{-\frac{1}{2}}\left(\breve{\Sigma}_{k+1|k}\breve{C}_{k+1}^T - \Psi\breve{\Sigma}_{k|k-1}\breve{C}_k^T\right) \\
&\quad = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} + \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} \\
&\quad = \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix}
\end{aligned}
$$

By this, $Y = \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} R_{e,k+1}^{-\frac{1}{2}} J_1$ is obtained.

Where, $\breve{C}_k^T = (\breve{C}_{k+1}\Psi)^T$.

(2,2)-Block Matrix

[0105]

$$
\begin{aligned}
-ZSZ^T + YJ_1Y^T & \\
&= \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix}^T - \rho^{-1}\tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \\
&= \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix}^T + \rho^{-1}\left(\breve{\Sigma}_{k+1|k} - \Psi \breve{\Sigma}_{k|k-1}\Psi^T\right) \\
&= \rho^{-1}\Psi\left(\begin{bmatrix} K_k \\ 0 \end{bmatrix} R_{e,k}^{-1} \begin{bmatrix} K_k \\ 0 \end{bmatrix}^T - \breve{\Sigma}_{k|k-1}\right)\Psi^T + \rho^{-1}\breve{\Sigma}_{k+1|k} \\
&= -\Psi\breve{\Sigma}_{k+1|k}\Psi^T + \breve{\Sigma}_{k+2|k+1} + \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} R_{e,k+1}^{-1} \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix}^T
\end{aligned}
$$

By this, $-ZSZ^T = \breve{\Sigma}_{k+2|k+1} - \Psi\breve{\Sigma}_{k+1|k}\Psi^T = -\tilde{L}_{k+1}R_{r,k+1}^{-\frac{1}{2}} S R_{r,k+1}^{-\frac{1}{2}} \tilde{L}_{k+1}^T$ and $Z = \tilde{L}_{k+1}R_{r,k+1}^{-\frac{1}{2}}$ is obtained.

8-3. Proof of Theorem 4

[0106] When an observation matrix H_k has a shift characteristic and

$$J = (J_1 \oplus - S),$$

the following relational expression is obtained by a similar method to Theorem 2.

$$
\begin{bmatrix} R_{e,k+1} & 0 \\ \begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} & \tilde{L}_{k+1} \end{bmatrix} = \begin{bmatrix} R_{e,k} & \breve{C}_{k+1}\tilde{L}_k \\ \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} & \rho^{-\frac{1}{2}}\tilde{L}_k \end{bmatrix} \Theta(k) \tag{46}
$$

[0107] Where,

$$\Theta(k) = (J_1 R_{e,k}^{\frac{1}{2}} \oplus -R_{r,k}^{\frac{1}{2}})\Sigma(k)(R_{e,k+1}^{-\frac{1}{2}} J_1^{-1} \oplus -R_{r,k+1}^{-\frac{1}{2}})$$

$$
\Sigma(k) = \begin{bmatrix} I & -R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \\ -R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T & I \end{bmatrix} \tag{47}
$$

[0108] Where, $\Sigma(k)^T(R_{e,k} \oplus -R_{r,k})\,\Sigma(k) = (R_{e,k+1} \oplus -R_{r,k+1})$ and $R_{r,k+1}$ is determined so that $\Sigma(k)^T (R_{e,k} \oplus - R_{r,k})\Sigma(k) = (R_{e,k+1} \oplus - R_{r,k+1})$ is established. Next, when an update expression of $R_{r,k+1}$ is newly added to the third line of expression (46), the following expression is finally obtained.

$$\begin{bmatrix} R_{e,k+1} & 0 \\ \begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} & \tilde{L}_{k+1} \\ 0 & R_{r,k+1} \end{bmatrix} = \begin{bmatrix} R_{e,k} & \breve{C}_{k+1}\tilde{L}_k \\ \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} & \rho^{-\frac{1}{2}}\tilde{L}_k \\ \tilde{L}_k^T\breve{C}_{k+1}^T & R_{r,k} \end{bmatrix} \begin{bmatrix} I & -R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \\ -R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T & I \end{bmatrix}$$

$$= \begin{bmatrix} R_{e,k} - \breve{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T & 0 \\ \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T & \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \\ 0 & R_{r,k} - \tilde{L}_k^T\breve{C}_{k+1}^T R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \end{bmatrix} \quad (48)$$

[0109] From the correspondence of the respective terms of 3 × 2 block matrixes of both sides, the following update expression of a gain matrix K^{-}_{k} is obtained.

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T \quad (49)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \quad (50)$$

$$R_{e,k+1} = R_{e,k} - \breve{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T \quad (51)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T\breve{C}_{k+1}^T R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k \quad (52)$$

Industrial Applicability

[0110] In general, in a normal civil communication equipment or the like, calculation is often performed at single precision in view of the cost and speed. Thus, as the practical state estimation algorithm, the present invention would have effects in various industrial fields. Besides, the invention can be applied to an echo canceller in a communication system or a sound system, sound field reproduction, noise control and the like.

Claims

1. A system estimation method, applicable to a system in the form of a communication system or a sound system or used for sound field reproduction or noise control, for making state estimation robust and optimizing a forgetting factor ρ simultaneously in an estimation algorithm, said system being defined by a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

where,

x_k is defined as a state vector,
w_k is defined as a system noise,
v_k is defined as an observation noise,
y_k is defined as an observation signal,
z_k is defined as an output signal,
F_k is defined as dynamics of said system, and
G_k is defined as a drive matrix,
H_k is defined as an observation matrix,

wherein as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise w_k and the observation noise v_k and is weighted with the forgetting factor ρ, is suppressed to be smaller than a term corresponding to a predetermined upper limit value γ_f, said filter error being associated with a filter used in said system estimation method, said filter receiving as an input signal said observation signal of said system, and
the system estimation method comprises:

a step at which a processing section used in said system estimation method inputs the upper limit value γ_f, the observation signal y_k as an input of said filter and a value including the observation matrix H_k from a storage section or an input section;
a step at which the processing section determines the forgetting factor ρ relevant to said system defined by the state space model in accordance with the upper limit value γ_f;
a step of executing a hyper H_∞ filtering in said filter at which the processing section reads out an initial value or a value including the observation matrix H_k at a time from the storage section and obtains a filter gain $K_{s,k}$ by using the forgetting factor ρ and a gain matrix K^-_k and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \tag{25}$$

$$K_{s,k} = \rho^{\frac{1}{2}} \overline{K}_k(:,1)/R_{e,k}(1,1) \tag{26}$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \tag{27}$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \tag{28}$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \tag{29}$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \tag{30}$$

where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\bar{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}} K_k \quad (31)$$

where,

y_k is the observation signal,
F_k is the dynamics of the system,
H_k is the observation matrix,
$x\hat{}_{k|k}$ is defined as the estimated value of the state vector x_k at the time k using the observation signals y_0 to y_k,
$K_{s,k}$ is defined as the filter gain, obtained from the gain matrix K^{-}_{k}, and
$R_{e,k}$, $L^{\sim}_{k}$ are defined as auxiliary variables;
a step at which the processing section stores an estimated value of the state vector x_k obtained by the hyper H_∞ filtering into the storage section;
a step at which the processing section calculates an existence condition based on the upper limit value γ_f and the forgetting factor ρ by the obtained observation matrix H_i or the observation matrix H_i and the filter gain $K_{s,i}$, wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \quad (18)$$

here,

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1 - H_i K_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \quad (19)$$

where the forgetting factor ρ and the upper limit value γ_f have a following relation:
$0 < \rho = 1 - \chi(\gamma_f) \leq 1$, where $\chi(\gamma_f)$ denotes a monotonically damping function of γ_f to satisfy $\chi(1) = 1$ and $\chi(\infty) = 0$, and
a step at which the processing section sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section, by decreasing the upper limit value γ_f and repeating the step of executing the hyper H_∞ filtering.

2. The system estimation method according to claim 1, wherein an estimated value $z^{v}_{k|k}$ of the output signal z_k is obtained from the estimated value $x\hat{}_{k|k}$ of the state vector x_k at the time k by a following expression:

$$z^{v}_{k|k} = H_k x\hat{}_{k|k}.$$

3. The system estimation method according to claim 1, wherein the H_∞ filter equation is applied to obtain the estimated value $x\hat{}_{k|k} = [h\hat{}_1[k], \cdots, h\hat{}_N[k]]^T$
a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots \tag{34}$$

and
an echo canceller is realized by canceling an actual echo by the obtained pseudo-echo.

4. A system estimation program for causing a computer to perform the method of any of claims 1 to 3.

5. A computer readable recording medium recording a system estimation program for causing a computer to perform the system estimation method of any of claims 1 to 3.

6. A system estimation device applicable to a system in the form of a communication system or a sound system or for sound field reproduction or noise control, for making state estimation robust and optimizing a forgetting factor ρ simultaneously in an estimation algorithm, said system being defined by
a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

where,

x_k is defined as a state vector,
w_k is defined as a system noise,
v_k is defined as an observation noise,
y_k is defined as an observation signal,
z_k is defined as an output signal,
F_k is defined as dynamics of a system,
G_k is defined as a drive matrix, and
H_k is defined as an observation matrix,

wherein, as an evaluation criterion, a maximum value of an energy gain which indicates a ratio of a filter error to a disturbance including the system noise w_k and the observation noise v_k and is weighted with the forgetting factor ρ is suppressed to be smaller than a term corresponding to a predetermined upper limit value γ_f, said filter error being associated with a filter implemented in said system estimation device, said filter receiving as an input signal said observation signal of said system, and
the system estimation device comprises:

a processing section configured to estimate said system by operating on said state space model; and
a storage section to which reading and/or writing is performed by the processing section and which stores respective observed values, set values, and estimated values relevant to said system defined by the state space model, wherein,
a first means configured to enable the processing section to input the upper limit value γ_f, the observation signal y_k as an input of said filter and a value including an observation matrix H_k from the storage section or an input section;
a second means configured to enable the processing section to determine the forgetting factor ρ relevant to said system defined by the state space model in accordance with the upper limit value γ_f;
a third means having implemented said filter so as to execute a hyper H_∞ filtering and being configured to enable

the processing section to read out an initial value or a value including the observation matrix H_k at a time from the storage section and to obtain a filter gain $K_{s,k}$ by using the forgetting factor ρ and a gain matrix K_k and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k\hat{x}_{k-1|k-1}) \tag{25}$$

$$K_{s,k} = \rho^{\frac{1}{2}}\overline{K}_k(:,1)/R_{e,k}(1,1) \tag{26}$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\check{C}_{k+1}^T \tag{27}$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{28}$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\check{C}_{k+1}^T \tag{29}$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T\check{C}_{k+1}^T R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{30}$$

Where,

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1\check{\Sigma}_{1|0}\check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}}K_k \tag{31}$$

here,

y_k is the observation signal,
F_k is the dynamics of the system,
H_k is the observation matrix,
$\hat{x}_{k|k}$ is defined as the estimated value of the state vector x_k at the time k using the observation signals y_0 to y_k,
$K_{s,k}$ is the filter gain, obtained from the gain matrix $\overline{K}_k$, and
$R_{e,k}$, $\tilde{L}_k$ are defined as auxiliary variables;
a fourth means configured to enable the processing section to store an estimated value of the state x_k obtained by the hyper H_∞ filtering into the storage section;
a fifth means configured to enable the processing section to calculate an existence condition based on the upper limit value γ_f and the forgetting factor ρ by the obtained observation matrix H_i or the observation matrix H_i and the filter gain $K_{s,i}$,

wherein the processing section calculates the existence condition in accordance with a following expression:

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0,\cdots,k \tag{18}$$

here,

$$\varrho = 1-\gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1-H_i K_{s,i}}, \quad \rho = 1-\chi(\gamma_f) \tag{19}$$

where the forgetting factor ρ and the upper limit value γ_f have a following relation: $0 < \rho = 1 - \chi(\gamma_f) \leq 1$, where $\chi(\gamma_f)$ denotes a monotonically damping function of γ_f to satisfy $\chi(1) = 1$ and $\chi(\infty) = 0$, and

a sixth means configured to enable the processing section to set the upper limit value to be small within a range where the existence condition is satisfied at each time and to store the value into the storage section, by decreasing the upper limit value γ_f and re-applying said the third means for executing the hyper H_∞ filtering.

Patentansprüche

1. Systemschätzverfahren, das auf ein System in der Form eines Kommunikationssystems oder einer Tonanlage anwendbar ist oder das für die akustische Feldwiedergabe oder die Rauschsteuerung verwendet wird, um gleichzeitig die Zustandsschätzung robust zu machen und einen Gedächtnisfaktor ρ in einem Schätzalgorithmus zu optimieren, wobei das System definiert ist durch:

ein Zustandsraummodell, das durch die folgenden Ausdrücke ausgedrückt wird:

$$x_{k+1} = F_k G_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

wobei

x_k als ein Zustandsvektor definiert ist,
w_k als ein Systemrauschen definiert ist,
v_k als ein Beobachtungsrauschen definiert ist,
y_k als ein Beobachtungssignal definiert ist,
z_k als ein Ausgangssignal definiert ist,
F_k als die Dynamik des Systems definiert ist, und
G_k als eine Ansteuermatrix definiert ist,
H_k als eine Beobachtungsmatrix definiert ist,

wobei als ein Bewertungskriterium ein Maximalwert einer Energieverstärkung, die ein Verhältnis eines Filterfehlers zu einer Störung anzeigt, die das Systemrauschen w_k und das Beobachtungsrauschen v_k umfasst und mit dem Gedächtnisfaktor ρ gewichtet wird, klein gehalten wird, um kleiner als ein Term zu sein, der einem vorgegebenen oberen Grenzwert γ_f entspricht, wobei der Filterfehler zu einem Filter gehört, das in dem Systemschätzverfahren verwendet wird, wobei das Filter das Beobachtungssignal des Systems als ein Eingangssignal empfängt, und das Systemschätzverfahren aufweist:

einen Schritt, in dem ein Verarbeitungsabschnitt, der in dem Systemschätzverfahren verwendet wird, den oberen Grenzwert γ_f, das Beobachtungssignal y_k als einen Eingang des Filters und einen Wert, der die Beobachtungsmatrix H_k aus einem Speicherabschnitt oder einem Eingangsabschnitt umfasst, einspeist;
einen Schritt, in dem der Verarbeitungsabschnitt den Gedächtnisfaktor ρ, der für das durch das Zustandsraummodell definierte System relevant ist, gemäß dem oberen Grenzwert γ_f bestimmt;
einen Schritt zum Ausführen einer Hyperfilterung H_∞ in dem Filter, wobei der Verarbeitungsabschnitt gleichzeitig einen Anfangswert oder einen Wert, der die Beobachtungsmatrix H_k umfasst, aus dem Speicherabschnitt ausliest und eine Filterverstärkung $K_{s,k}$ unter Verwendung des Gedächtnisfaktors ρ und einer Verstärkungsmatrix K^{-}_{k} und durch folgende Ausdrücke gewinnt:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k\hat{x}_{k-1|k-1}) \tag{25}$$

$$K_{s,k} = \rho^{\frac{1}{2}}\overline{K}_k(:,1)/R_{e,k}(1,1) \tag{26}$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T \check{C}_{k+1}^T \tag{27}$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{28}$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\check{C}_{k+1}^T \tag{29}$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T\check{C}_{k+1}^T R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{30}$$

wobei hier

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix},\quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k],\quad \check{H}_1 = [u(1),0,\ldots,0]$$
$$R_{e,1} = R_1 + \check{C}_1\check{\Sigma}_{1|0}\check{C}_1^T,\quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix},\quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2,\rho^3,\ldots,\rho^{N+2}\},\quad \rho = 1-\chi(\gamma_f)$$
$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2},\quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix},\quad \overline{K}_0 = 0,\quad \hat{x}_{0|0} = \check{x}_0,\quad \overline{K}_k = \rho^{-\frac{1}{2}}K_k \tag{31}$$

wobei

y_k das Beobachtungssignal ist,

F_k die Dynamik des Systems ist,
H_k die Beobachtungsmatrix ist,
$x^\wedge_{k|k}$ als der Schätzwert für den Zustandsvektor x_k zu der Zeit k unter Verwendung von Beobachtungssignalen y_0 bis y_k definiert ist,
$K_{s,k}$ als die Filterverstärkung definiert ist und aus der Verstärkungsmatrix K^-_k erhalten wird,
$R_{e,k}$, $L^\sim_k$ als Hilfsvariable definiert sind;

einen Schritt, in dem der Verarbeitungsabschnitt einen Schätzwert des Zustandsvektors x_k, der durch die Hyperfilterung H_∞ erhalten wird, in den Speicherabschnitt speichert;
einen Schritt, in dem der Verarbeitungsabschnitt eine Existenzbedingung basierend auf dem oberen Grenzwert γ_f und dem Gedächtnisfaktor ρ durch die erhaltene Beobachtungsmatrix H_i oder die Beobachtungsmatrix H_i und die Filterverstärkung $K_{s,i}$ berechnet,
wobei der Verarbeitungsabschnitt die Existenzbedingung gemäß einem folgenden Ausdruck berechnet:

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \tag{18}$$

wobei hier

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1 - H_i K_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \tag{19}$$

wobei der Gedächtnisfaktor p und der obere Grenzwert γ_f die folgende Beziehung haben:
$0 < \rho = 1 - \chi(\gamma_f) \leq 1$, wobei $\chi(\gamma_f)$ eine monotone Dämpfungsfunktion von γ_f bezeichnet, so dass $\chi(1) = 1$ und $\chi(\infty) = 0$ erfüllt sind, und
einen Schritt, in dem der Verarbeitungsabschnitt den oberen Grenzwert derart festlegt, dass er innerhalb eines Bereichs, in dem die Existenzbedingung zu jeder Zeit erfüllt ist, klein ist, und den Wert in den Speicherabschnitt speichert, indem er den oberen Grenzwert γ_f verringert und den Schritt der Ausführung der Hyperfilterung H_∞ wiederholt.

2. Systemschätzverfahren nach Anspruch 1, wobei ein Schätzwert $z^v_{k|k}$ des Ausgangssignals z_k aus Schätzwert $x^\wedge_{k|k}$ des Zustandsvektors x_k zu der Zeit k durch einen folgenden Ausdruck erhalten wird:

$$z^v_{k|k} = H_k x^\wedge_{k|k}.$$

3. Systemschätzverfahren nach Anspruch 1, wobei die H_∞-Filtergleichung angewendet wird, um den Schätzwert $x^\wedge_{k|k} = [h^\wedge_1[k], ..., h^\wedge_N[k]]^T$ zu erhalten,
wobei ein Pseudoecho durch einen folgenden Ausdruck geschätzt wird:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots \tag{34}$$

und
ein Echokompensator realisiert wird, indem ein tatsächliches Echo durch das erhaltene Pseudoecho kompensiert wird.

4. Systemschätzprogramm, um zu bewirken, dass ein Computer das Verfahren nach einem der Ansprüche 1 bis 3 durchführt.

5. Computerlesbares Aufzeichnungsmedium, das ein Systemschätzprogramm aufzeichnet, um zu bewirken, dass ein Computer das Systemschätzverfahren nach einem der Ansprüche 1 bis 3 durchführt.

6. Systemschätzvorrichtung, die auf ein System in der Form eines Kommunikationssystems oder einer Tonanlage oder für die akustische Feldwiedergabe oder die Rauschsteuerung anwendbar ist, um gleichzeitig die Zustandsschätzung robust zu machen und einen Gedächtnisfaktor ρ in einem Schätzalgorithmus zu optimieren, wobei das System definiert ist durch:

ein Zustandsraummodell, das durch die folgenden Ausdrücke ausgedrückt wird:

$$x_{k+1} = F_k G_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

wobei

x_k als ein Zustandsvektor definiert ist,
w_k als ein Systemrauschen definiert ist,
v_k als ein Beobachtungsrauschen definiert ist,
y_k als ein Beobachtungssignal definiert ist,
z_k als ein Ausgangssignal definiert ist,
F_k als die Dynamik des Systems definiert ist,
G_k als eine Ansteuermatrix definiert ist, und
H_k als eine Beobachtungsmatrix definiert ist,

wobei als ein Bewertungskriterium ein Maximalwert einer Energieverstärkung, die ein Verhältnis eines Filterfehlers zu einer Störung anzeigt, die das Systemrauschen w_k und das Beobachtungsrauschen v_k umfasst und mit dem Gedächtnisfaktor p gewichtet wird, klein gehalten wird, um kleiner als ein Term zu sein, der einem vorgegebenen oberen Grenzwert γ_f entspricht, wobei der Filterfehler zu einem Filter gehört, das in der Systemschätzvorrichthung implementiert ist, wobei das Filter das Beobachtungssignal des Systems als ein Eingangssignal empfängt, und
die Systemschätzvorrichtung aufweist:

einen Verarbeitungsabschnitt, der konfiguriert ist, um das System durch Arbeiten mit dem Zustandsraummodell zu schätzen; und
einen Speicherabschnitt, in dem durch den Verarbeitungsabschnitt Lesen und/oder Schreiben durchgeführt wird und der jeweilige Beobachtungswerte, Sollwerte und Schätzwerte, die für das durch das Zustandsraummodell definierte System relevant sind, speichert, wobei
eine erste Einrichtung, die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt den oberen Grenzwert γ_f, das Beobachtungssignal y_k als einen Eingang des Filters und einen Wert, der eine Beobachtungsmatrix H_k umfasst, aus einem Speicherabschnitt oder einem Eingangsabschnitt einspeist;
eine zweite Einrichtung, die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt den Gedächtnisfaktor ρ, der für das durch das Zustandsraummodell definierte System relevant ist, gemäß dem oberen Grenzwert γ_f bestimmt;
eine dritte Einrichtung, die das Filter implementiert hat, um eine Hyperfilterung H_∞ auszuführen, und die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt gleichzeitig einen Anfangswert oder einen Wert, der die Beobachtungsmatrix H_k umfasst, aus dem Speicherabschnitt ausliest und eine Filterverstärkung $K_{s,k}$ unter Verwendung des Gedächtnisfaktors p und einer Verstärkungsmatrix K^-_k und durch folgende Ausdrücke gewinnt:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k\hat{x}_{k-1|k-1}) \tag{25}$$

$$K_{s,k} = \rho^{\frac{1}{2}}\overline{K}_k(:,1)/R_{e,k}(1,1) \tag{26}$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T \check{C}_{k+1}^T \tag{27}$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{28}$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T \check{C}_{k+1}^T \tag{29}$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \tag{30}$$

wobei hier

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1\check{\Sigma}_{1|0}\check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}}K_k \tag{31}$$

wobei

y_k das Beobachtungssignal ist,
F_k die Dynamik des Systems ist,
H_k die Beobachtungsmatrix ist,
$x^{\wedge}_{k|k}$ als der Schätzwert für den Zustandsvektor x_k zu der Zeit k unter Verwendung von Beobachtungssignalen y_0 bis y_k definiert ist,
$K_{s,k}$ die Filterverstärkung ist die aus der Verstärkungsmatrix K^{-}_k erhalten wird, und
$R_{e,k}$, $L^{\sim}_k$ als Hilfsvariable definiert sind;

eine vierte Einrichtung, die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt einen Schätzwert des Zustands x_k, der durch die Hyperfilterung H_∞ erhalten wird, in den Speicherabschnitt speichert;
eine fünfte Einrichtung, die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt eine Existenz-

bedingung basierend auf dem oberen Grenzwert γ_f und dem Gedächtnisfaktor ρ durch die erhaltene Beobachtungsmatrix H_i oder die Beobachtungsmatrix H_i und die Filterverstärkung $K_{s,i}$ berechnet,
wobei der Verarbeitungsabschnitt die Existenzbedingung gemäß einem folgenden Ausdruck berechnet:

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \tag{18}$$

wobei hier

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho \boldsymbol{H}_i \boldsymbol{K}_{s,i}}{1 - \boldsymbol{H}_i \boldsymbol{K}_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \tag{19}$$

wobei der Gedächtnisfaktor p und der obere Grenzwert γ_f die folgende Beziehung haben:
$0 < \rho = 1 - \chi(\gamma_f) \leq 1$, wobei $\chi(\gamma_f)$ eine monotone Dämpfungsfunktion von γ_f bezeichnet, so dass $\chi(1) = 1$ und $\chi(\infty) = 0$ erfüllt sind, und
eine sechste Einrichtung, die konfiguriert ist, um zu ermöglichen, dass der Verarbeitungsabschnitt den oberen Grenzwert derart festlegt, dass er innerhalb eines Bereichs, in dem die Existenzbedingung zu jeder Zeit erfüllt ist, klein ist, und den Wert in den Speicherabschnitt speichert, indem der obere Grenzwert γ_f verringert wird und die dritte Einrichtung zum Ausführen der Hyperfilterung H_∞ erneut angewendet wird.

Revendications

1. Procédé d'évaluation de système applicable à un système sous la forme d'un système de communication ou d'un système de sonorisation ou utilisé pour la reproduction de champ sonore ou la lutte contre le bruit, pour rendre robuste une évaluation d'état dudit système et optimiser un facteur d'oubli ρ, simultanément, dans un algorithme d'évaluation, ledit système étant défini par un modèle d'espace d'état exprimé par les expressions suivantes :

$$x_{k+1} = F_k x_k + G_K w_K$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

dans lesquelles

x_k est défini comme un vecteur d'état,
w_k est défini comme un bruit de système,
v_k est défini comme un bruit d'observation,
y_k est défini comme un signal d'observation,
z_k est défini comme un signal de sortie,
F_k est défini comme une dynamique dudit système, et
G_k est défini comme une matrice de commande,
H_k est défini comme une matrice d'observation,

étant précisé que comme critère d'évaluation, une valeur maximale d'un gain d'énergie qui indique un rapport d'une erreur de filtre sur une perturbation comprenant le bruit de système w_k et le bruit d'observation v_k et qui est pondérée avec le facteur d'oubli p, est supprimée pour être plus petite qu'un terme correspondant à une valeur limite supérieure γ_f prédéterminée, l'erreur de filtre étant associée à un filtre utilisé dans le procédé d'évaluation de système, le filtre

recevant comme signal d'entrée le signal d'observation du système, et
le procédé d'évaluation de système comprend :

une étape pendant laquelle la section de traitement utilisée dans le procédé d'évaluation de système entre la valeur limite supérieure γ_f, le signal d'observation γ_f comme entrée du filtre, et une valeur contenant la matrice d'observation H_k à partir d'une section de stockage ou d'une section d'entrée ;
une étape pendant laquelle la section de traitement détermine le facteur d'oubli p concernant ledit système défini par le modèle d'espace d'état selon la valeur limite supérieure γ_f ;
une étape d'exécution d'un filtrage hyper H_∞ dans le filtre, lors de laquelle la section de traitement extrait de la section de stockage une valeur initiale ou une valeur contenant la matrice d'observation H_k à un moment, et obtient un gain de filtre $K_{s,k}$ en utilisant le facteur d'oubli p et une matrice de gain K''_k et les expressions suivantes :

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k\hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}}\overline{K}_k(:,1)/R_{e,k}(1,1) \quad (26)$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}}\tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1}\tilde{L}_k R_{r,k}^{-1}\tilde{L}_k^T\check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T\check{C}_{k+1}^T R_{e,k}^{-1}\check{C}_{k+1}\tilde{L}_k \quad (30)$$

dans lesquelles

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1}\ u(k+1-N)] = [u(k+1)\ u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$
$$R_{e,1} = R_1 + \check{C}_1\check{\Sigma}_{1|0}\check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$
$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{k}{2}}K_k \quad (31)$$

dans lesquelles

y_k est le signal d'observation,
F_k est la dynamique du système,

H_k est la matrice d'observation,
$x^{\wedge}_{k|k}$ est défini comme la valeur évaluée du vecteur d'état x_k au moment k utilisant les signaux d'observation y_o à y_k,
$K_{s,k}$ est défini comme le gain de filtre, obtenu à partir de la matrice de gain K''_k, et
$R_{s,k}$, L''_k sont définis comme des variables auxiliaires ;

une étape lors de laquelle la section de traitement stocke dans la section de stockage une valeur évaluée du vecteur d'état x_k obtenu par le filtrage hyper H_∞ ;
une étape lors de laquelle la section de traitement calcule une condition d'existence sur la base de la valeur limite supérieure γ_f et du facteur d'oubli φ par la matrice d'observation H_i obtenue ou la matrice d'observation H_i et le gain de filtre $K_{s,i}$,
étant précisé que la section de traitement calcule la condition d'existence selon une expression suivante :

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \tag{18}$$

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1 - H_i K_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \tag{19}$$

dans laquelle le facteur d'oubli p et la valeur limite supérieure γ_f ont une relation suivante :
$O < \rho = 1 - \chi(\gamma_f) \leq 1$, où $\chi(\gamma_f)$ indique une fonction d'amortissement monotone de γ_f pour satisfaire $\chi(1) = 1$ et $\chi(\infty) = 0$, et
une étape lors de laquelle la section de traitement fixe la valeur limite supérieure pour qu'elle soit faible à l'intérieur d'une plage dans laquelle la condition d'existence est satisfaite à tout moment, et stocke la valeur dans la section de stockage, en réduisant la valeur limite supérieure γ_f et en répétant l'étape d'exécution du filtrage hyper H-.

2. Procédé d'évaluation de système selon la revendication 1, étant précisé qu'une valeur évaluée $z^{v}_{k|k}$ du signal de sortie est obtenue à partir de la valeur évaluée $x^{\wedge}_{k|k}$ du vecteur d'état x_k au moment k par une expression suivante :

$$z^{v}_{k|k} = H_k x^{\wedge}_{k|k}.$$

3. Procédé d'évaluation de système selon la revendication 1, étant précisé que l'équation de filtre H_∞ est appliquée pour obtenir la valeur évaluée $X^{\wedge}_{k|k}=[h^{\wedge}_1[k],...,h^{\wedge}_N[k]]^T$
qu'un pseudo-écho est évalué par une expression suivante :

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{(k-i)}, \quad k = 0, 1, 2, \cdots \tag{34}$$

et qu'un suppresseur d'écho est réalisé par la suppression d'un écho réel par le pseudo-écho obtenu.

4. Programme d'évaluation de système pour amener un ordinateur à mettre en oeuvre le procédé de l'une quelconque des revendications 1 à 3.

5. Support d'enregistrement lisible par ordinateur, enregistrant un programme d'évaluation de système pour amener un ordinateur à mettre en oeuvre le procédé de l'une quelconque des revendications 1 à 3.

6. Dispositif d'évaluation de système applicable à un système sous la forme d'un système de communication ou d'un système de sonorisation ou utilisé pour la reproduction de champ sonore ou la lutte contre le bruit, pour rendre robuste une évaluation d'état dudit système et optimiser un facteur d'oubli p, simultanément, dans un algorithme d'évaluation, ledit système étant défini par un modèle d'espace d'état exprimé par les expressions suivantes :

$$x_{k+1} = F_k x_k + G_K w_K$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

dans lesquelles

x_k est défini comme un vecteur d'état,
w_k est défini comme un bruit de système,
v_k est défini comme un bruit d'observation,
y_k est défini comme un signal d'observation,
z_k est défini comme un signal de sortie,
F_k est défini comme une dynamique dudit système, et
G_k est défini comme une matrice de commande,
H_k est défini comme une matrice d'observation,

étant précisé que comme critère d'évaluation, une valeur maximale d'un gain d'énergie qui indique un rapport d'une erreur de filtre sur une perturbation comprenant le bruit de système w_k et le bruit d'observation v_k et qui est pondérée avec le facteur d'oubli p, est supprimée pour être plus petite qu'un terme correspondant à une valeur limite supérieure γ_f prédéterminée, l'erreur de filtre étant associée à un filtre utilisé dans le procédé d'évaluation de système, le filtre recevant comme signal d'entrée le signal d'observation du système, et
le dispositif d'évaluation de système comprend :

une section de traitement pour évaluer ledit système en fonctionnant sur le modèle d'espace d'état ; et
une section de stockage dans laquelle la lecture et/ou l'écriture sont réalisées par la section de traitement, et qui stocke les valeurs observées respectives, les valeurs fixées et les valeurs évaluées concernant le système défini par le modèle d'espace d'état,
des premiers moyens conçus pour permettre à la section de traitement d'entrer la valeur limite supérieure γ_f, le signal d'observation y_k comme entrée du filtre et une valeur contenant une matrice d'observation H_k à partir de la section de stockage ou d'une section d'entrée ;
des deuxièmes moyens conçus pour permettre à la section de traitement de déterminer le facteur d'oubli p concernant le système défini par le modèle d'espace d'état selon la valeur limite supérieure γ_f ;
des troisièmes moyens dans lesquels le filtre est installé de manière à exécuter un filtrage hyper H_∞, et qui sont conçus pour permettre à la section de traitement d'extraire de la section de stockage une valeur initiale ou une valeur contenant la matrice d'observation H_k à un moment, et pour obtenir un gain de filtre $K_{s,k}$ en utilisant le facteur d'oubli p et une matrice de gain K''_k et les expressions suivantes :

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:,1)/R_{e,k}(1,1) \quad (26)$$

$$\begin{bmatrix} \overline{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \qquad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \overline{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \qquad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \qquad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \qquad (30)$$

dans lesquelles

$$\check{C}_{k+1} = \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \; u(k+1-N)] = [u(k+1) \; u_k], \quad \check{H}_1 = [u(1), 0, \ldots, 0]$$

$$R_{e,1} = R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \mathrm{diag}\{\rho^2, \rho^3, \ldots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f)$$

$$\tilde{L}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1)\times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \overline{K}_0 = 0, \quad \hat{x}_{0|0} = \check{x}_0, \quad \overline{K}_k = \rho^{-\frac{1}{2}} K_k \qquad (31)$$

dans lesquelles

y_k est le signal d'observation,
F_k est la dynamique du système,
H_k est la matrice d'observation,
$x^{\wedge}_{k|k}$ est défini comme la valeur évaluée du vecteur d'état x_k au moment k utilisant les signaux d'observation y_o à y_k,
$K_{s,k}$ est défini comme le gain de filtre, obtenu à partir de la matrice de gain K''_k, et
$R_{s,k}$, L''_k sont définis comme des variables auxiliaires ;

des quatrièmes moyens conçus pour permettre à la section de traitement de stocker dans la section de stockage une valeur évaluée de l'état x_k obtenu par le filtrage hyper H_∞ ;
des cinquièmes moyens de stockage conçus pour permettre à la section de traitement de calculer une condition d'existence sur la base de la valeur limite supérieure γ_f et du facteur d'oubli φ par la matrice d'observation H_i obtenue ou la matrice d'observation H_i et le gain de filtre $K_{s,i}$,
étant précisé que la section de traitement calcule la condition d'existence selon une expression suivante :

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \cdots, k \qquad (18)$$

$$\varrho = 1-\gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1-H_i K_{s,i}}, \quad \rho = 1-\chi(\gamma_f) \tag{19}$$

dans laquelle le facteur d'oubli p et la valeur limite supérieure γ_f ont une relation suivante :
O < p = 1 - $\chi(\gamma_f) \leq 1$, où $\chi(\gamma_f)$ indique une fonction d'amortissement monotone de γ_f pour satisfaire $\chi(1) = 1$ et $\chi(\infty) = 0$, et

des sixièmes moyens conçus pour permettre à la section de traitement de fixer la valeur limite supérieure pour qu'elle soit faible à l'intérieur d'une plage dans laquelle la condition d'existence est satisfaite à tout moment, et de stocker la valeur dans la section de stockage, en réduisant la valeur limite supérieure γ_f et en réappliquant les troisièmes moyens pour exécuter le filtrage hyper H_∞.

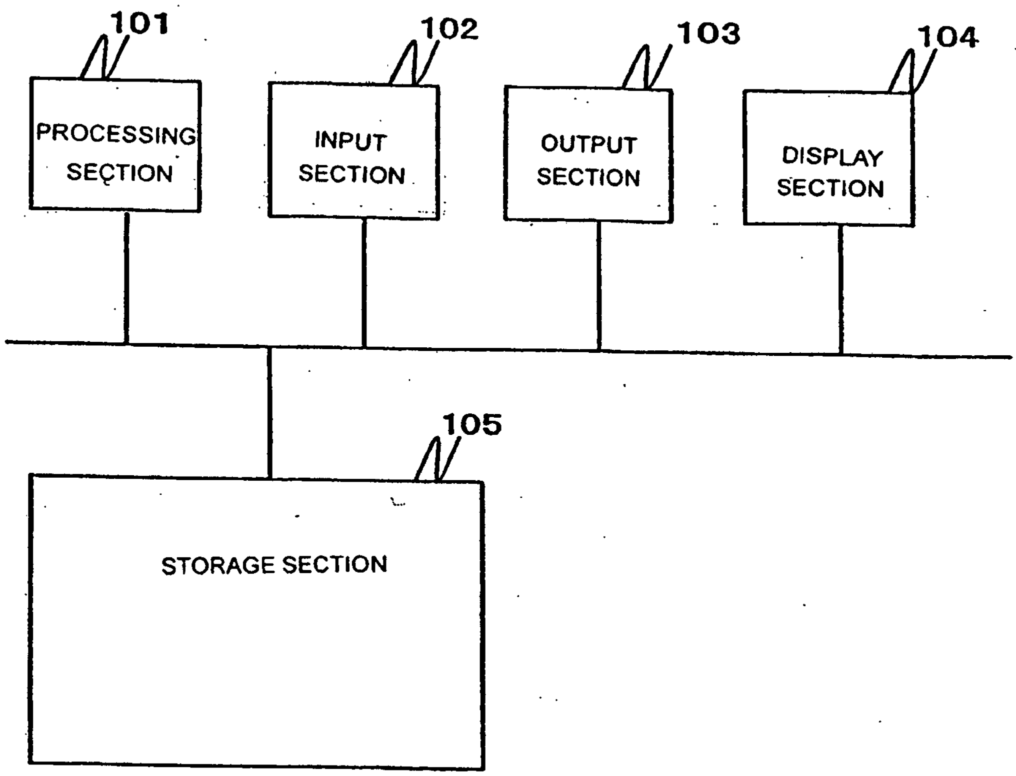

101
PROCESSING SECTION
102
INPUT SECTION
103
OUTPUT SECTION
104
DISPLAY SECTION
105
STORAGE SECTION

FIG. 1

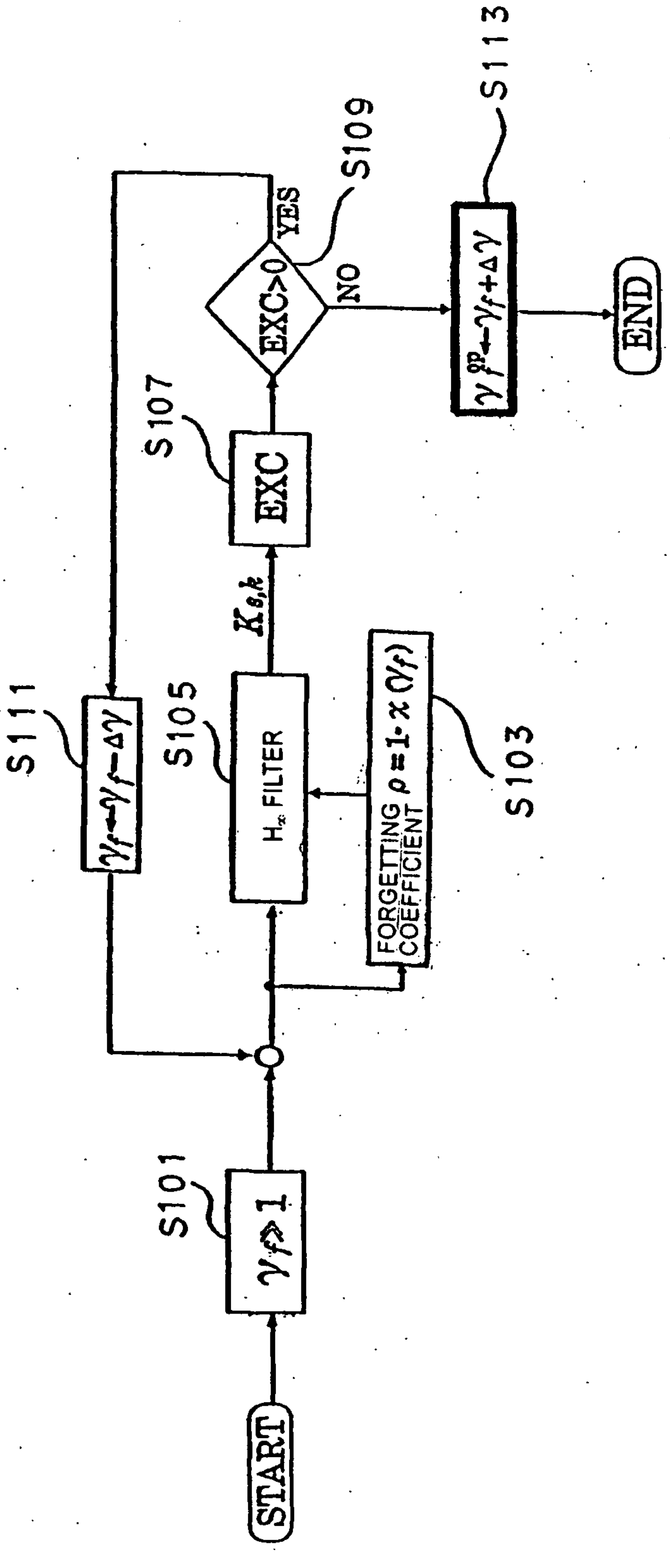

START
S101
γf>1
S111
γf←γf−Δγ
S105
H∞ FILTER
FORGETTING COEFFICIENT ρ=1-χ(γf)
S103
Ks,k
S107
EXC
S109
EXC>0
YES
NO
S113
γf^op←γf+Δγ
END

FIG. 2

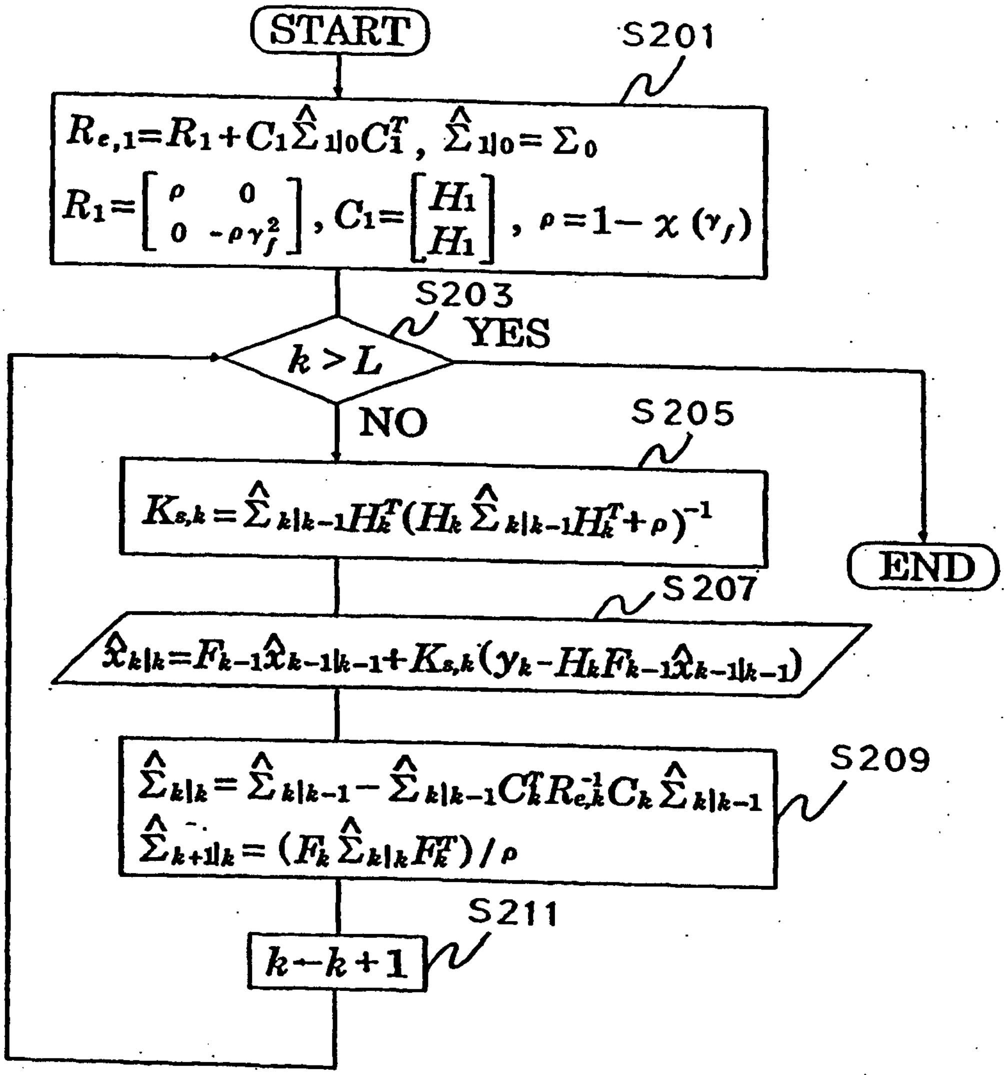

START
S201
$R_{e,1}=R_1+C_1\hat{\Sigma}_{1|0}C_1^T,\ \hat{\Sigma}_{1|0}=\Sigma_0$
$R_1=\begin{bmatrix}\rho & 0\\ 0 & -\rho\gamma_f^2\end{bmatrix},\ C_1=\begin{bmatrix}H_1\\ H_1\end{bmatrix},\ \rho=1-\chi(\gamma_f)$
S203
$k>L$
YES
NO
S205
$K_{s,k}=\hat{\Sigma}_{k|k-1}H_k^T(H_k\hat{\Sigma}_{k|k-1}H_k^T+\rho)^{-1}$
END
S207
$\hat{x}_{k|k}=F_{k-1}\hat{x}_{k-1|k-1}+K_{s,k}(y_k-H_kF_{k-1}\hat{x}_{k-1|k-1})$
S209
$\hat{\Sigma}_{k|k}=\hat{\Sigma}_{k|k-1}-\hat{\Sigma}_{k|k-1}C_k^TR_{e,k}^{-1}C_k\hat{\Sigma}_{k|k-1}$
$\hat{\Sigma}_{k+1|k}=(F_k\hat{\Sigma}_{k|k}F_k^T)/\rho$
S211
$k\leftarrow k+1$

FIG. 3

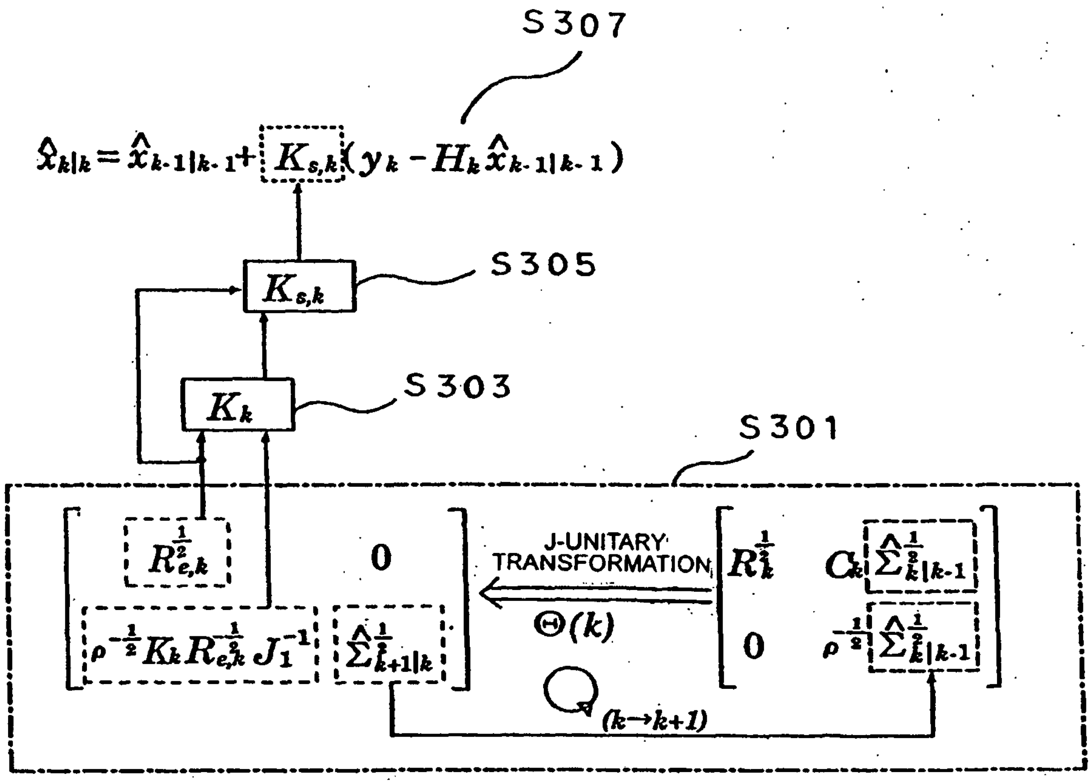


FIG. 4

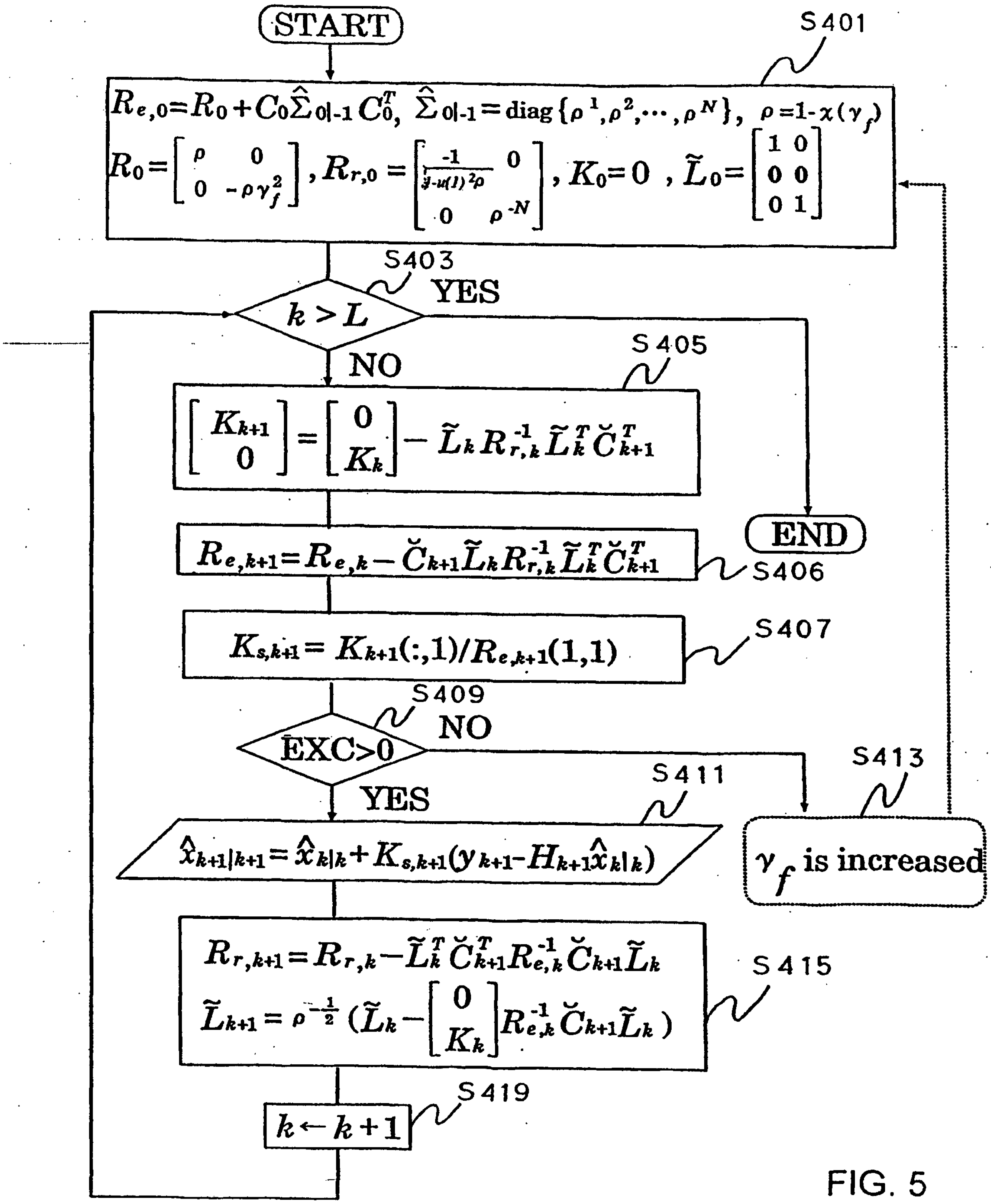
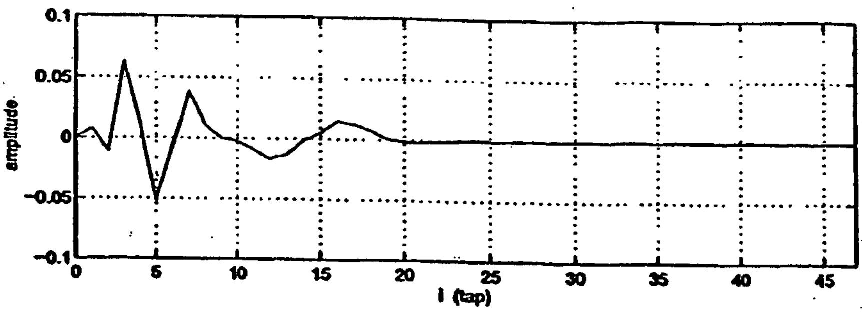

START
S401
$R_{e,0}=R_0+C_0\hat{\Sigma}_{0|-1}C_0^T,\ \hat{\Sigma}_{0|-1}=\mathrm{diag}\{\rho^1,\rho^2,\cdots,\rho^N\},\ \rho=1-\chi(\gamma_f)$
$R_0=\begin{bmatrix}\rho & 0\\ 0 & -\rho\gamma_f^2\end{bmatrix},R_{r,0}=\begin{bmatrix}\frac{-1}{1-u(1)^2\rho} & 0\\ 0 & \rho^{-N}\end{bmatrix},K_0=0\ ,\tilde{L}_0=\begin{bmatrix}1&0\\0&0\\0&1\end{bmatrix}$
S403
$k>L$
YES
NO
S405
$\begin{bmatrix}K_{k+1}\\0\end{bmatrix}=\begin{bmatrix}0\\K_k\end{bmatrix}-\tilde{L}_kR_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T$
END
$R_{e,k+1}=R_{e,k}-\breve{C}_{k+1}\tilde{L}_kR_{r,k}^{-1}\tilde{L}_k^T\breve{C}_{k+1}^T$
S406
$K_{s,k+1}=K_{k+1}(:,1)/R_{e,k+1}(1,1)$
S407
S409
EXC>0
NO
S413
YES
S411
$\hat{x}_{k+1|k+1}=\hat{x}_{k|k}+K_{s,k+1}(y_{k+1}-H_{k+1}\hat{x}_{k|k})$
γ_f is increased
$R_{r,k+1}=R_{r,k}-\tilde{L}_k^T\breve{C}_{k+1}^TR_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k$
$\tilde{L}_{k+1}=\rho^{-\frac{1}{2}}(\tilde{L}_k-\begin{bmatrix}0\\K_k\end{bmatrix}R_{e,k}^{-1}\breve{C}_{k+1}\tilde{L}_k)$
S415
S419
$k\leftarrow k+1$

FIG. 5

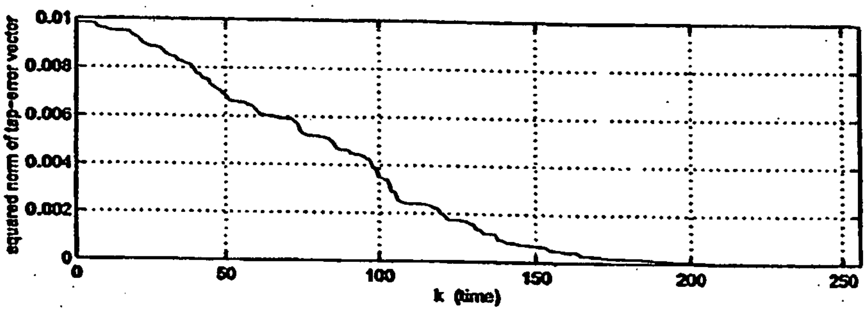
IMPULSE RESPONSE OF ECHO PATH

h_0	h_1	h_2	h_3	h_4	h_5
0.0	0.008	-0.012	0.064	0.013	-0.052
h_6	h_7	h_8	h_9	h_{10}	h_{11}
-0.007	0.039	0.011	0.0	-0.002	-0.009
h_{12}	h_{13}	h_{14}	h_{15}	h_{16}	h_{17}
-0.016	-0.013	-0.001	0.004	0.015	0.013
h_{18}	h_{19}	h_{20}	h_{21}	h_{22}	h_{23}
0.007	0.0	-0.001	-0.002	-0.001	0.0

FIG. 6

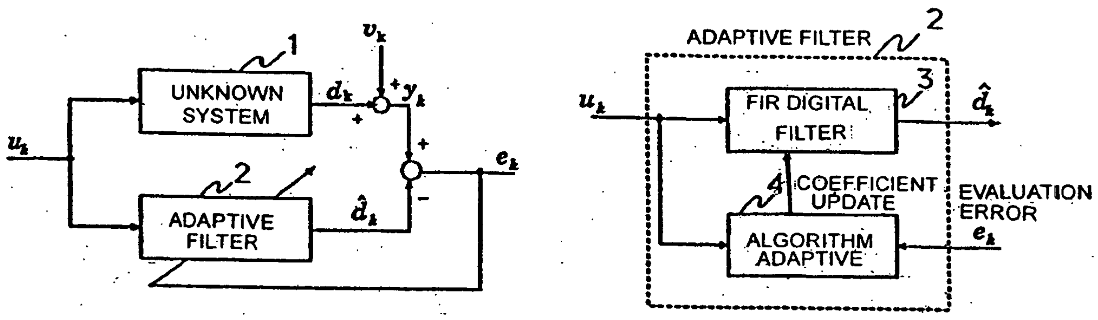


(a) ESTIMATED VALUE OF IMPULSE RESPONSE



(b) TRANSITION OF TAP ERROR

FIG. 7



(a) OUTPUT ERROR SYSTEM

(b) STRUCTURE OF ADAPTIVE FILTER

FIG. 8

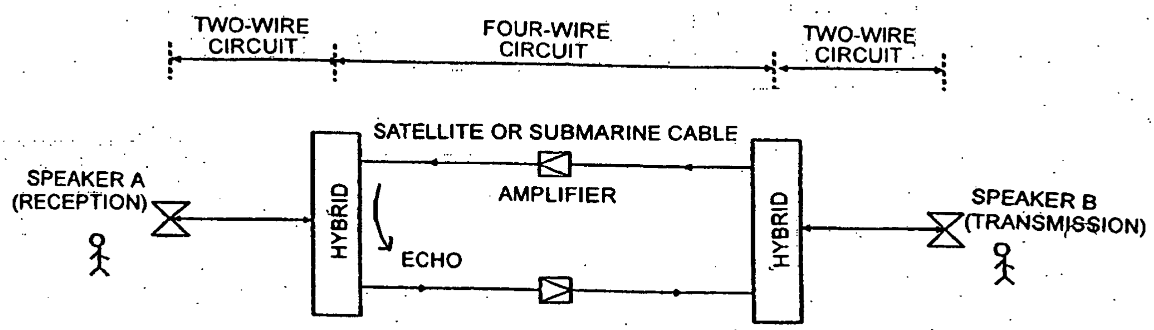

TWO-WIRE CIRCUIT
FOUR-WIRE CIRCUIT
TWO-WIRE CIRCUIT
SATELLITE OR SUBMARINE CABLE
SPEAKER A (RECEPTION)
HYBRID
AMPLIFIER
ECHO
HYBRID
SPEAKER B (TRANSMISSION)

FIG. 9

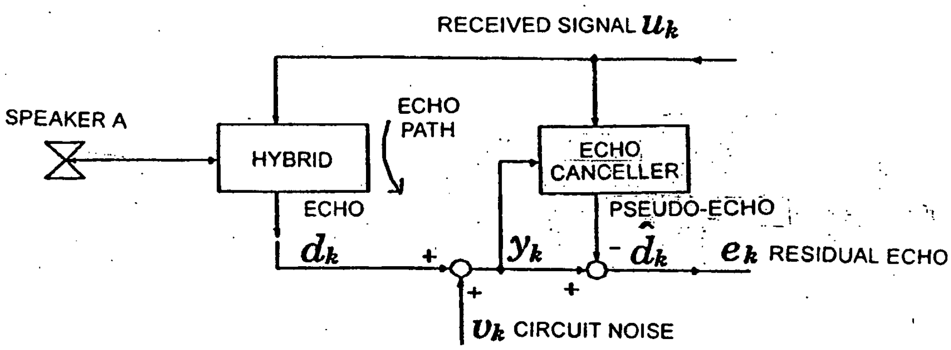

RECEIVED SIGNAL u_k
SPEAKER A
HYBRID
ECHO PATH
ECHO CANCELLER
ECHO
PSEUDO-ECHO
d_k
+
y_k
$-\hat{d}_k$
e_k RESIDUAL ECHO
+
+
v_k CIRCUIT NOISE

FIG. 10

REFERENCES CITED IN THE DESCRIPTION

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- **B. HASSIBI ; A. H. SAYED ; T. KAILATH.** Indefinite-Quadratic Estimation and Control. *SIAM,* 1996 **[0016]**
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- An H∞ Optimization and Its Fast Algorithm for Time-Variant System Identification. **NISHIYAMA K.** IEEE TRANSACTIONS ON SIGNAL PROCESSING. IEEE SERVICE CENTER, 01 May 2004, vol. 52, 1335-1342 **[0020]**
- **NISHIYAMA K.** FAST J-UNITARY ARRAY FORM OF THE HYPER H∞ FILTER. *IEICE TRANSACTIONS ON FUNDAMENTALS OF ELECTRONICS, COMMUNICATIONS AND COMPUTER SCIENCES, ENGINEERING SCIENCES SOCIETY,* 01 November 2005, vol. E88-A (11), ISSN 0916-8508, 3143-3150 **[0021]**