Application of spatial filter techniques to neuromagnetic source localization

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1. Introduction

My talk will focus on a particular class of MEG source localization techniques referred to as the spatial filter.The spatial filter can provide the spatio-temporal reconstruction of neural activities from MEG measurements without assuming any kind of source model nor a large number of discrete sources assigned to voxels. Two kinds of spatial filter techniques exist: one is non-adaptive and the other is adaptive. In the conventional non-adaptive technique, its spatial resolution has an intrinsic limitation imposed by a hardware configuration. An adaptive technique, however, can attain a spatial resolution that exceeds this hardware limitation. After reviewing these properties of spatial filter techniques, my talk will particularly focus on our proposed adaptive vector spatial filter technique, and will show that it can reconstruct spatio-temporal pattern of neural activities with a several-milli-meter spatial resolution and with a one-milli-second temporal resolution.

2. Spatial filter formulation

Let us define the magnetic field measured by the mth sensor at time t as $b_m(t)$, and a column vector $\boldsymbol{b}(t) = [b_1(t), b_2(t), \ldots, b_M(t)]^T$ as a set of measured data where M is the total number of sensors and the superscript T indicates the matrix transpose. A spatial location (x, y, z) is represented by a three-dimensional vector $\boldsymbol{r}$: $\boldsymbol{r} = (x, y, z)$. The source moment magnitude at $\boldsymbol{r}$ and time t is defined as a three-dimensional vector $\boldsymbol{s}(\boldsymbol{r}, t)$. The estimate of the source moment is denoted as $\hat{\boldsymbol{s}}(\boldsymbol{r}, t)$. We define the lead field vector for the jth sensor as $\boldsymbol{l}_j(\boldsymbol{r}) = [l_j^x(\boldsymbol{r}), l_j^y(\boldsymbol{r}), l_j^z(\boldsymbol{r})]$. Here, $l_j^x(\boldsymbol{r})$, $l_j^y(\boldsymbol{r})$, and $l_j^z(\boldsymbol{r})$ are the jth sensor readings caused when a single source exists at $\boldsymbol{r}$ with the unit moment directed in the x, y, and z directions, respectively. This lead field vector $\boldsymbol{l}_j(\boldsymbol{r})$ represents the sensitivity of the jth sensor. We define the lead field matrix, which represents the sensitivity of the whole sensor array, as $\boldsymbol{L}(\boldsymbol{r})$. Here, its jth row is equal to $\boldsymbol{l}_j(\boldsymbol{r})$.

Using these definitions, the fundamental relationship between the measured magnetic field and the source moment are obtained such that

$$\boldsymbol{b}(t) = \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r}, t)d\boldsymbol{r}. \tag{1}$$

The problem of the source localization is the problem of obtaining the best estimate of $\boldsymbol{s}(\boldsymbol{r}, t)$ from the array measurement $\boldsymbol{b}(t)$. Let us define a set of weight that characterizes the property of the spatial filter as a column vector $\boldsymbol{w}(\boldsymbol{r}) = [w_1(\boldsymbol{r}), w_2(\boldsymbol{r}), \ldots, w_M(\boldsymbol{r})]^T$. The spatial filter technique reconstructs the source moment by applying the following simple linear operation,

$$\hat{s}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = \sum_{m=1}^{M} w_m(\boldsymbol{r})b_m(t). \tag{2}$$

3. Resolution kernel

Combining Equations (1) and (2), we obtain the relationship,

$$\hat{s}(\boldsymbol{r},t) = \int \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r}')s(\boldsymbol{r}',t)d\boldsymbol{r}' = \int \mathbb{R}(\boldsymbol{r},\boldsymbol{r}')s(\boldsymbol{r}',t)d\boldsymbol{r}'. \tag{3}$$

Here, $\mathbb{R}(\boldsymbol{r},\boldsymbol{r}') = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r}')$, and this $\mathbb{R}(\boldsymbol{r},\boldsymbol{r}')$ is often referred to as the resolution kernel, which describes the relationship between the original and estimated source distributions. The major problem with spatial filter techniques is how to derive a set of appropriate weight. Therefore, we need a criterion that evaluate the appropriateness of the filter weight. The resolution kernel can provide this criterion. In other words, the weight must be chosen so that the resolution kernel has an appropriate shape. First of all, the kernel should be peaked at $\boldsymbol{r}$. Only in this case, the reconstructed source distribution can be interpreted as a smoothed version of the true source distribution. However, if this condition is not met, The reconstructed source distribution may be totally different from the true source distribution. The kernel should also have a small main-lobe width, and low side-lobe amplitude to have higher spatial resolution and less artifact contamination.

4. Non-adaptive spatial filter

Non-adaptive spatial filter uses a weight that is independent from the measurement. Probably, the most well-known spatial filter of this kind is the minimum-norm estimate [1][2]. The weight can be obtained by the following minimization,

$$\min \int ||\mathbb{R}(\boldsymbol{r},\boldsymbol{r}') - \delta(\boldsymbol{r}-\boldsymbol{r}')||^2 d\boldsymbol{r}'. \tag{4}$$

That is, by trying to make the resolution kernel close to the delta-function, the weight is obtained and expressed as

$$\boldsymbol{w}^T(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{G}^{-1}. \tag{5}$$

Here an element of this matrix $\boldsymbol{G}$ is given by calculating the overlap between the lead fields of sensors,

$$G_{i,j} = \int \boldsymbol{l}_i(\boldsymbol{r})\boldsymbol{l}_j(\boldsymbol{r})d\boldsymbol{r}$$

Unfortunately, in biomagnetic instruments, the overlaps between the adjacent sensor lead fields are large, and $G_{i,j}$ has a more or less similar value for various pairs of i and j. Consequently, the matrix $\boldsymbol{G}$ is generally very poorly conditioned. This fact greatly affects the performance of this non-adaptive spatial filter method because it requires to calculate the inverse of $\boldsymbol{G}$, and this process should be very erroneous if $\boldsymbol{G}$ is nearly singular.

5. Adaptive spatial filter

An adaptive spatial filter technique uses a data-dependent weight. Because adaptive spatial filter techniques do not need to use the inversion of $\boldsymbol{G}$, they generally give a performance which is not restricted by the property of $\boldsymbol{G}$. In other words, an adaptive technique can attain the performance which exceeds the hardware limitations.

5.1 Minimum-variance estimator

The most well-known adaptive filter technique is the minimum-variance estimator [3]. In this method, the spatial filter weight is obtained by solving the constrained optimization problem,

$$\min \boldsymbol{w}^T \boldsymbol{D} \boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{\eta}). \tag{6}$$

and we have

$$\boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta}) = \frac{\boldsymbol{l}^T(\boldsymbol{r}, \boldsymbol{\eta}) \boldsymbol{D}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r}, \boldsymbol{\eta}) \boldsymbol{D}^{-1} \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{\eta})} \tag{7}$$

Here, $\boldsymbol{D}$ is the covariance matrix of the measurement, the vector $\boldsymbol{\eta}$ represents the predetermined orientation, and $\boldsymbol{l}(\boldsymbol{r}, \boldsymbol{\eta})$ is defined as $\boldsymbol{l}(\boldsymbol{r}, \boldsymbol{\eta}) = \boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}$. The idea behind the above optimization is that the filter weight is designed to minimize the total output signal power with maintaining the signal from the pointing location. Therefore, ideally, this weight only passes the signal from the pointing location $\boldsymbol{r}$, and suppresses the signals from other locations.

5.2 Proposed vector-type adaptive spatial filter

The minimum-variance estimator has been developed in the field of seismic, radar and sonar array processing. When applying this technique to the MEG source localization problem, several MEG-specific problems arise. We propose a spatial filter technique which can overcome these problems [4]. The derivation of the proposed spatial filter weight is twofold. Let us define the unit column vectors in the x, y, and z directions as $\boldsymbol{f}_x$, $\boldsymbol{f}_y$, and $\boldsymbol{f}_z$. The weight for our proposed spatial filter in a intermediate stage is derived as

$$\bar{\boldsymbol{w}}_\mu = \frac{\boldsymbol{D}^{-1}\boldsymbol{L}(\boldsymbol{r})[\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{f}_\mu}{\sqrt{\boldsymbol{f}_\mu^T \boldsymbol{\Omega} \boldsymbol{f}_\mu}}, \tag{8}$$

where $\mu = x, y$, or z, and

$$\boldsymbol{\Omega} = [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{D}^{-2}\boldsymbol{L}(\boldsymbol{r})[\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}.$$

Defining $\boldsymbol{E}_S$ as a matrix whose columns consist of the signal-level eigenvectors of $\boldsymbol{D}$, the final weight vectors for the proposed spatial filter are derived by projecting the weight vector $\bar{\boldsymbol{w}}_\mu$ onto the signal subspace of the measurement covariance matrix,

$$\boldsymbol{w}_\mu = \boldsymbol{E}_S \boldsymbol{E}_S^T \bar{\boldsymbol{w}}_\mu. \tag{9}$$

Here, the weight vector $\boldsymbol{w}_\mu$ detects each μ component of the source moment (where $\mu = x, y$, or z), so the weight matrix $[\boldsymbol{w}_x, \boldsymbol{w}_y, \boldsymbol{w}_z]$ gives the estimate of the source moment vector $\hat{\boldsymbol{s}}(\boldsymbol{r}, t)$.

6. Experiments and results

The non-adaptive minimum-norm spatial filter technique and our proposed adaptive spatial filter technique were applied to various types of MEG data sets. The results of these applications confirm the following:

1. Our adaptive spatial filter technique can provide a spatial resolution better than that of the non adaptive technique.
2. This is because the spatial resolution for the non-adaptive technique is limited by the sensor configuration but the adaptive technique can exceed this limit (the super resolution can be attained).
3. Correlated source activities, however, affect the quality of the results obtained by the adaptive technique.

Therefore, the adaptive technique may be suited to observe relatively small cortical regions with a high spatial resolution, and the non-adaptive technique may be suited to simultaneously observe the whole-brain activities.

References

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